

Analyze and Solve Systems of Linear Equations

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TOPIC
5

ANALYZE AND SOLVE SYSTEMS OF LINEAR EQUATIONS

? Topic Essential Question

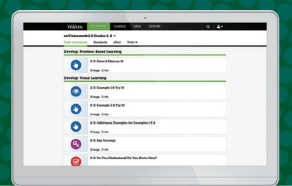
What does it mean to solve a system of linear equations?

Topic Overview

- 5-1 Estimate Solutions by Inspection
- 5-2 Solve Systems by Graphing
- 5-3 Solve Systems by Substitution
- 5-4 Solve Systems by Elimination
- 3-Act Mathematical Modeling: Ups and Downs

Topic Vocabulary

- solution of a system of linear equations
- system of linear equations



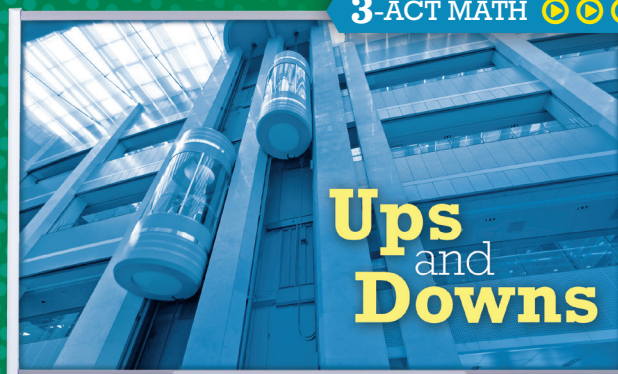
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Lesson Digital Resources

- INTERACTIVE ANIMATION** Interact with visual learning animations.
- ACTIVITY** Use with *Solve & Discuss It*, *Explore It*, and *Explain It* activities, and to explore Examples.
- VIDEOS** Watch clips to support 3-Act Mathematical Modeling Lessons and STEM Projects.
- PRACTICE** Practice what you've learned.

3-ACT MATH



Ups and Downs

Ups and Downs

The express elevators at One World Trade Center in New York are some of the fastest in the world. They can take you 1,280 feet to the observation deck in 60 seconds. That's about 23 miles per hour! Compare that to a typical elevator that travels between 3 and 5 miles per hour. Think about this during the 3-Act Mathematical Modeling lesson.



Additional Digital Resources

- TUTORIALS** Get help from *Virtual Nerd*, right when you need it.
- KEY CONCEPT** Review important lesson content.
- GLOSSARY** Read and listen to English/Spanish definitions.
- ASSESSMENT** Show what you've learned.

- MATH TOOLS** Explore math with digital tools.
- GAMES** Play Math Games to help you learn.
- ETEXT** Interact with your Student's Edition online.

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Topic Essential Question

What does it mean to solve a system of linear equations?

Revisit the Topic Essential Question throughout the topic. See the Teacher's Edition for the Topic Review for notes about answering the question.

3-Act Mathematical Modeling

Have students read about the Mathematics Modeling lesson for this topic. You can use the preview for this lesson to get students interested in learning the content of this topic.

The Mathematical Modeling in 3 Acts lesson appears after Lesson 5-4.



Video



Activity

3-ACT MATH

3-Act Mathematical Modeling: Ups and Downs

Lesson Overview

Objectives

Students will be able to:

- ✓ use mathematical modeling to represent a problem situation and to propose a solution.
- ✓ test and verify the appropriateness of their math models.
- ✓ explain why the results from their mathematical models may not align exactly to the problem situation.

Essential Understanding

Many real-world problem situations can be represented with a mathematical model, but that model may not represent a real-world situation exactly.

Earlier in this topic, students:

- solved systems of linear equations using different methods.

In this lesson, students:

- develop a mathematical model to represent and propose a solution to a problem situation involving a system of linear equations.

Later in this course, students will:

- refine their mathematical modeling skills.

This mathematical modeling lesson focuses on **application** of both **math content** and **math practices** and **processes**.

- Students draw on their understanding of equality concepts to develop a representative model.
- Students apply their mathematical model to test and validate its applicability to similar problem situations.

Math Anytime



Today's Challenge

Use the Topic 5 problems any time during this topic.

The screenshot shows a digital interface for 'Today's Challenge'. It includes a 'Go back' button, a progress bar for 'Day 1' through 'Day 5' (with 'Day 1' selected), and a '2 of 7' indicator. The main content area displays a 'Model' problem: 'Write an equation that gives the maximum heart rate, y , for a person who is x years old. Graph the equation. Explain the meaning of the y -intercept and slope.' To the right of the problem is a table titled 'Heart Rates (beats per minute)'.

Heart Rates (beats per minute)	
Maximum Heart Rate (MHR)	$220 - \text{age}$
Vigorous Intensity Exercise	70–85% of MHR
Moderate Intensity Exercise	50–70% of MHR



Mathematics Overview

In this lesson, students will develop and use a mathematical model to represent and propose a solution to a real-world problem involving a system of linear equations. Students will reinforce both their procedural skills and their understanding of equality concepts, while recognizing the limitations of some mathematical models for real-world situations.

Applying Math Practices

Model with Mathematics

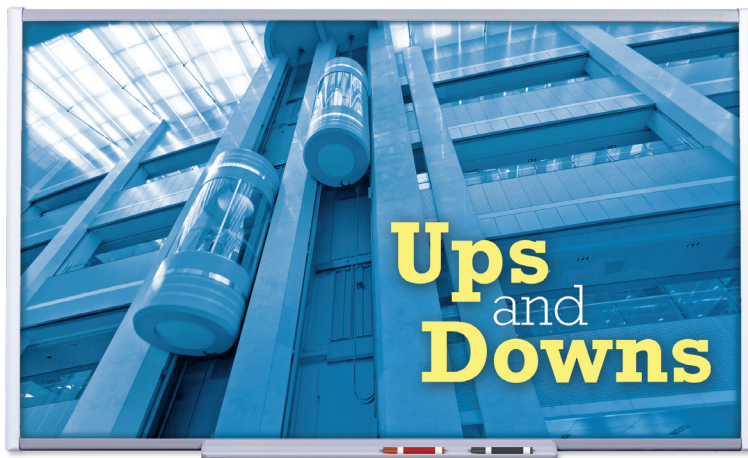
The focus of this lesson is on mathematical modeling. Students identify the relationship among variables, develop a model that represents the situation, and use the model to propose a solution. Students interpret their solutions and propose explanations for why their answers may not match the real-world answer.

As students carry out mathematical modeling, they will also engage in sense-making, abstract and quantitative reasoning, and mathematical communication and argumentation. In testing and validating their models, students look for patterns and structure.



3-Act Mathematical Modeling

ACT 1 The Hook



Students will be tasked with determining whether taking the stairs or taking the elevator is faster.

Play the Video and Brainstorm Questions

Have students complete **Question 1**. Encourage them to consider the situation and ask any questions that arise. Listen for interesting mathematical and non-mathematical questions. Ask students what makes each question interesting.

Q: What questions do you have? [Sample questions: Where are they going? Why did one of them take the stairs? How many floors are they traveling?]

Pose the Main Question

After the question brainstorming, pose the Main Question students will be tasked with answering. Have students complete **Question 2**.

Main Question

Q: Which route is faster?

Ask about Predictions

Have students complete **Questions 3 and 4**. You can survey the class for the range of predictions.

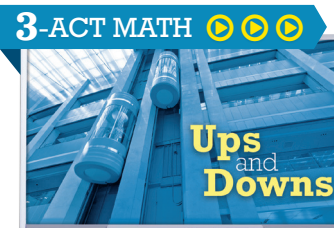
Q: Why do you think your prediction is the answer to the Main Question?

Q: Who had a similar prediction?

Q: How many of you agree with that prediction?

Q: Who has a different prediction?

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3-Act Mathematical Modeling:
Ups and Downs

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ACT 1

1. After watching the video, what is the first question that comes to mind?

2. Write the Main Question you will answer.



3. Make a prediction to answer this Main Question.

The person who wins took the .

4. **Construct Arguments** Explain how you arrived at your prediction.

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3-Act Mathematical Modeling *continued*



Activity

ACT 2 The Model



Identify Important Info

Have students complete **Questions 5**.

Q: What information would be helpful to solve the problem?

[Sample answers: How quickly the person walks down stairs; how long the other person waits for the elevator; what the rate of the elevator is; how many floors they travel]

Q: How could you get that information?

Q: Why do you need that information?

Reveal the Information

Reveal the information provided below using the online interactivity. Have students record information in **Question 6**.

Floors:

1 floor = 12.5 feet

Elevator

Rate: 125 feet per minute

Wait time: 38 seconds

Stairs

Rate: 1 floor, 8 seconds

Develop a Model

As students answer **Questions 7 and 8**, look for different representations they may use to model the situation. They might solve a system of equations, or examine the intersection point on a graph.

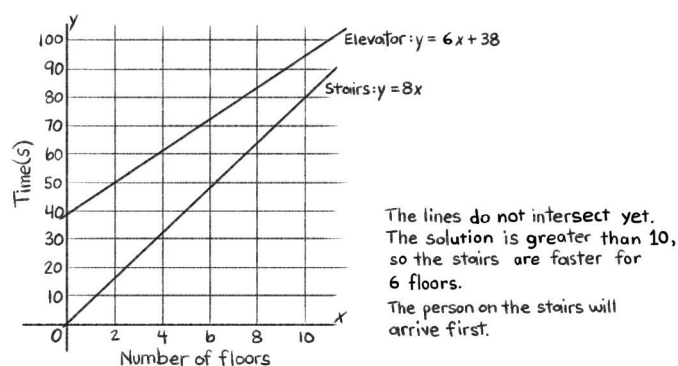
Q: Compare the rates of change and initial values. [Sample answer: The rate of change is greater for the stairs. The initial value for the stairs is 0, and the initial value for the elevator is 38.]

Use the Model to Propose a Solution

After students answer **Questions 7 and 8**, facilitate a discussion about solution methods. If needed, project the possible student solutions (shown below).

Possible Student Solutions

Sasha's Work



Sasha graphs both equations and estimates the solution. She uses the values of the equations on either side of the intersection point to answer the question.

Hunter's Work

$$\begin{aligned} \frac{125 \text{ feet}}{60 \text{ seconds}} &= \frac{12.5 \text{ feet}}{x \text{ seconds}} \\ x &= 6 \\ y &= 6x + 38 \\ y &= 8x \\ \text{Substitute } 8x \text{ for } y. \\ 8x &= 6x + 38 \\ 2x &= 38 \\ x &= 19 \end{aligned}$$

They take the same amount of time for 19 floors. The person in the elevator doesn't pass the person on the stairs.

Hunter also writes a system of equations and then uses the substitution method to solve for x . He interprets the meaning of the intersection point and uses rates to answer the question.

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ACT 2

5. What information in this situation would be helpful to know? How would you use that information?

6. **Use Appropriate Tools** What tools can you use to get the information you need? Record the information as you find it.

7. **Model with Math** Represent the situation using the mathematical content, concepts, and skills from this topic. Use your representation to answer the Main Question.

8. What is your answer to the Main Question? Does it differ from your prediction? Explain.



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Video

ACT 3 The Solution and Sequel



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ACT 3

9. Write the answer you saw in the video.



10. **Reasoning** Does your answer match the answer in the video? If not, what are some reasons that would explain the difference?

11. **Make Sense and Persevere** Would you change your model now that you know the answer? Explain.



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ACT 3 Extension

Reflect

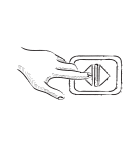
12. **Model with Math** Explain how you used a mathematical model to represent the situation. How did the model help you answer the Main Question?



13. **Reason Abstractly** A classmate solved the problem using equations with independent variable a and dependent variable b . What do these variables represent in the situation?

SEQUEL

14. **Generalize** Write an equation or inequality to represent all numbers of flights for which the elevator is faster.



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Use the Video to Reveal the Answer

The final part of the video shows the entire trip for both friends. Have students complete **Question 9**. Congratulate the students who were closest to the actual answer.

Main Question Answer

The person who arrives first took the stairs.

Validate Conclusions

After students complete **Questions 10 and 11**, encourage them to discuss possible sources of error inherent in using math to model real-world situations. Look for students to point out that their models are still useful even though they are not perfect.

Q: Why does your answer not match the answer in the video?

[Sample answer: Neither rate is exactly constant. The elevator has to speed up and slow down, for example.]

Q: How useful was your model at predicting the answer?

Q: How could your model better represent the situation?

Reflect on Thinking

Reason Abstractly If time allows, have students complete **Questions 12 and 13** as an extension. Use this opportunity to discuss how students incorporate mathematical processes during the task.

Pose the Sequel

Generalize Use **Question 14** to present a similar problem situation involving solving systems of equations. You can assign to early finishers or as homework so students can test the usefulness of their models.

Q: Write an equation or inequality to represent all numbers of flights for which the elevator is faster.

Using their models and the answer in the video, look for students to use an inequality, such as $x > 6$ or $x \geq 7$.

Q: Would you expect the same answer if they both went up 6 floors? Explain. [Sample answer: No; it takes longer to walk up a flight of stairs than down one.]