



TOPIC 4

Mathematical Modeling in 3 Acts:
The Snack Shack

Lesson Overview

FOCUS

Objective

Students will be able to:

- ✓ Use mathematical modeling to represent a problem situation and to propose a solution.
- ✓ Test and verify the appropriateness of their math models.
- ✓ Explain why the results from their mathematical models might not align exactly with the problem situation.

Essential Understanding

Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly.

COHERENCE

Earlier in this topic, students:

- Solved radical equations.

In this lesson, students:

- Develop a mathematical model to represent and propose a solution to a problem situation involving radical functions.

Later in this topic, students will:

- Refine their mathematical modeling skills using function operations and inverse relations and functions.

RIGOR

This mathematical modeling lesson focuses on application of both math content and math practices and processes.

- Students draw on their understanding of concepts related to radical functions to develop a representative model.
- Students apply their mathematical model to test and validate its applicability to similar problem situations.

**MATHEMATICAL
MODELING
IN 3 ACTS**

Video

The Snack Shack

Americans seem to love the beach! When the weather is warm, people flock to the beach. Some people bring coolers packed with food and drinks. Others prefer to take advantage of snack bars and shops set up along the beach.

Some beachside communities have built long wooden walkways, or boardwalks, to make it easier for beachgoers to walk to the snack bars and stores. How easy do you find walking in the sand? Think about this during the Mathematical Modeling in 3 Acts lesson.

ACT 1 Identify the Problem

1. What is the first question that comes to mind after watching the video?
2. Write down the main question you will answer about what you saw in the video.
3. Make an initial conjecture that answers this main question.
4. Explain how you arrived at your conjecture.
5. What information will be useful to know to answer the main question? How can you get it? How will you use that information?

ACT 2 Develop a Model

6. Use the math that you have learned in this Topic to refine your conjecture.

ACT 3 Interpret the Results

7. Did your refined conjecture match the actual answer exactly? If not, what might explain the difference?

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Mathematics Overview

The Importance of Modeling

Rather than presenting a problem with all the necessary information, diagrams, and graphs in advance, this task also asks students to participate in the formulation of a problem.

Students decide what questions are interesting, what quantities are important, and how they could access information. They choose which math concepts are relevant to the task, using approximations or assumptions in the solution when necessary.

Finally, the task includes an actual, real-world answer. Students discuss possible sources of error inherent in using math to model real-world situations. Then they evaluate the usefulness of their models in the situation, improving them if necessary.

Mathematical Practices

Model with Mathematics

To solve the problem presented, students identify variables and the relationship among them, develop a model that represents the situation, and use the model to propose a solution. Students interpret their solutions and propose explanations for why their answer may not match the real-world answer.

Other Practices

Students also engage in sense-making, perseverance, and abstract and quantitative reasoning as they complete the task. In testing their models, students look for patterns in their models and use that structure to find a solution.



Video

TOPIC 4 Mathematical Modeling in 3 Acts

The Snack Shack

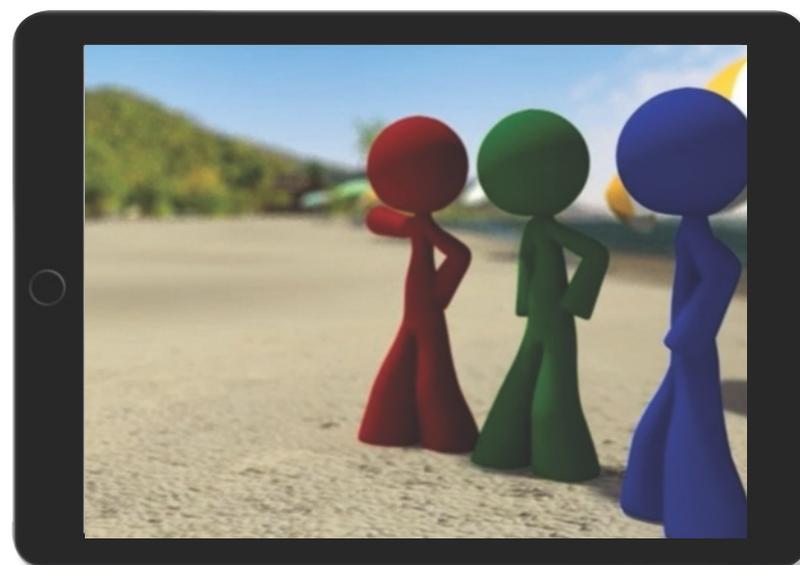
In this mathematical modeling task, students will explore and apply concepts related to radical expressions. Students will analyze a situation that involves finding the quickest way to a destination. They will be tasked with writing a mathematical model for each person and then simplifying radical expressions to answer the question posed. To do so, they apply concepts that they study in Topic 5.

ACT 1 The Hook

Play the video. The video shows three friends at the beach, who each take a different path to the snack shack. After the question brainstorming, present to students the Main Question they will be tasked with answering. Remind students to write down their questions and conjectures.

MAIN QUESTION

Who will reach the snack shack first?



ACT 2 Modeling With Math

Think about the task. Ask students to speculate how they could determine who will get to the snack shack first. Then have them think about what information they would need to find the answer.

Reveal the information. Use the video to give students information about the path each friend takes and their walking rates on the different surfaces. The information is also shown at the right.

What's the connection? Give students time to struggle as they think about how their ideas are connected to what they learned in this topic about radical expressions. How can they use variables to represent the distances? How can they model mathematically the time it takes each friend to reach the snack shack? Challenge them to simplify each radical expression completely before using their calculators to approximate.

INTERESTING MOMENTS WITH STUDENTS

Students will need to be precise when calculating times. Part of the challenge of this task is making sure students connect the right portion of each path to the right rate. If you notice students write down incorrect expressions for time, you can ask other students to invent a scenario where that expression would be the right answer.

Necessary Information

Walking rates

on sand: $\sqrt{4.5}$ ft/s

on boardwalk: 4.5 ft/s

Paths

Logan's route: $\sqrt{(5x)^2 + (7x)^2}$ ft on sand

Narrator's route: 5x ft on sand to the boardwalk, 7x ft on the main boardwalk

Olivia's route: $\sqrt{(2x)^2 + x^2}$ ft on sand to the side boardwalk, 4x feet on the side boardwalk, and 9x ft on the main boardwalk

The friends start at the same time and have the same walking rate.



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ACT 3 The Solution

Play the video. The final video shows the friends making their trips to the snack shack. The fastest route is revealed. Offer praise to the students whose conjectures are closest to the actual answer.

MAIN QUESTION ANSWER

The narrator will arrive first, after about 2 minutes, 37 seconds.

Do the “post-game” analysis. Ask students what the narrator meant when she said all three of them were right. Since the students’ times were within 5 seconds of each other, no one route was significantly faster than the others.

ONE POSSIBLE SOLUTION

Logan walks $\sqrt{(5x)^2 + (7x)^2}$ ft on sand. So it will take $\frac{\sqrt{(5x)^2 + (7x)^2}}{\sqrt{4.5}} = \frac{2x\sqrt{37}}{3}$, or about $4.06x$ seconds.

The narrator walks $5x$ ft on sand and $7x$ ft on the boardwalk. So it will take $\frac{5x}{\sqrt{4.5}} + \frac{7x}{4.5} = \frac{10x\sqrt{4.5} + 14x}{9}$, or about $3.91x$ seconds.

Olivia walks $\sqrt{(2x)^2 + x^2}$ ft on sand, and $5x + 9x$ ft on the boardwalk. It will take $\frac{x\sqrt{5}}{\sqrt{4.5}} + \frac{13x}{4.5} = \frac{2x\sqrt{22.5} + 26x}{9}$, or about $3.94x$ seconds.

The narrator will arrive first.

INTERESTING MOMENTS WITH STUDENTS

Why is there a variable? Students may notice that the posts are an equal distance apart, but the distance between posts is unknown. Using a variable means writing all three times in terms of x , but you can still compare the coefficients to find out whose path was fastest.

SEQUEL

As students finish, ask them how they can use the times in the Act 3 video to determine the length of each route. [Since students have calculated how long it takes each friend to walk to the snack shack in terms of x , they can set each expression equal to the number of seconds. They should find that x , the distance between posts, is about 40 ft. Then, substitute 40 for x in each of the three expressions to find the distances traveled.]

