SAVVAS

Research into Practice:

MATHEMATICS

Counting and Cardinality

BY DR. JUANITA COPLEY

The students in my kindergarten class were completing “I Can” books near the end of the school year. Five-year-old, Eli, wrote this beautiful entry expressing his excitement at being able to count to 30. As his teacher, I admit that I was equally excited about his progress because it was the result of many practice sessions. Indeed, he could count to 30 accurately without missing one word or object.

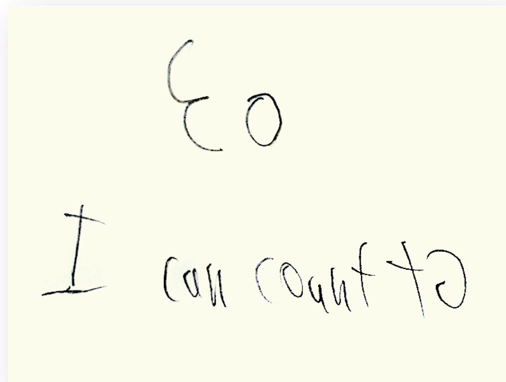

Figure 1. Eli’s “I Can” book entry, “I can count to 30.”

As these short examples illustrate, projects are an excellent way to connect students’ interests with high-quality mathematics curriculum and real-world events. If appropriately implemented, projects provide opportunities for the application of mathematics knowledge and skills.

No one questions that counting is an essential component of mathematical understanding. For example, *Counting and Cardinality* is a major domain for kindergartners as described by many state standards. In addition, counting objects or verbally saying the counting numbers has historically been foundational to early math learning. Both prekindergarten and kindergarten teachers name counting as one of three math ideas or procedures that children need to learn to “be ready” for first grade. For a young child, counting is not a simple process. Counting involves complex concepts, procedures that are modeled frequently, and the necessary practice to count successfully.

In this paper, we will examine five principles in counting and cardinality. While we will talk about each of these ideas separately, they are actually all connected to each other and are necessary for mathematical tasks. The five principles are: **(1)** cardinality (including subitizing), **(2)** object counting, **(3)** the verbal number word list, **(4)** reading and writing numbers, and **(5)** cardinal counting.

1. Cardinality

Cardinality is expressing the quantity of a set with one number or answering a “how many” question with only one number. This one number can be stated after objects have been counted individually. For example, a student might say, “One, two, three, four, five, six. There are 6 objects.” Cardinality can also be expressed by quickly recognizing the amount of objects without counting. For example, after viewing 3 objects outside a circle and 2 objects inside the circle, the counter simply states, “There are 5 objects.” In other words, the cardinal number is the last number said as the objects in the set are counted and it also refers to all of the objects in the set.

Most state standards are explicit in regards to cardinality. For example, here is a Kindergarten Mathematics standard about cardinality:

> Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

Subitizing is the term used to describe the act of quickly recognizing the cardinality of a small set of objects. Although not mentioned specifically in most state standards, research (NRC 2009; Dougherty et al. 2010; Clements and Sarama 2009; Baroody 2004) along with other curricular documents (Head Start 2018) advocates that subitizing be an important part of cardinality.

There are several terms used to describe subitizing, and these terms identify a partial developmental continuum.

- Perceptual subitizing: identifying cardinalities of 1, 2, and 3 without counting
- Conceptual subitizing: identifying cardinalities of 4 and 5 without counting by naming the parts of 4 and 5
- Extending subitizing 6–10: identifying cardinalities of 6, 7, 8, 9, and 10 as they relate to 5, e.g., *There are 8 objects because there are 3 more than 5 and that is 8.*
- Extending subitizing 11–20: identifying teen numbers as one group of 10 and a group of ones, e.g., *There are 17 objects because there is one group of 10 and another group of 7.*

HOW DO YOUNG CHILDREN DEMONSTRATE THEIR UNDERSTANDING OR THEIR PARTIAL UNDERSTANDING OF CARDINALITY?

Prekindergartener Alex was asked to count a set of 5 objects. He counted them accurately by saying 1, 2, 3, 4, 5 and touching each object as he said the counting sequence, demonstrating an understanding of one-to-one correspondence. When he was asked "how many" objects, he again repeated the counting sequence and did not answer with "5." Finally, when asked again, he repeated the same sequence and showed me 5 fingers on one hand. NOTE: Alex did not yet fully understand the concept of cardinality. Even when he showed me his 5 fingers, he wiggled each one as he counted, 1, 2, 3, 4, 5. When I said, "You have 5 objects," he repeated the word "five" after me but did not have the concept of cardinality.

In other words, Alex knew that he needed to count when asked the "how many" question. He did not know that the answer to the question is the last number said in the counting sequence.

When Kamaria, a kindergartner, was asked to count a group of 6 objects placed on a five-frame, she quickly said there were 6 objects altogether. When I asked her how she knew, she said there was 1 more than 5, so the total was 6. NOTE: Kamaria was able to extend her ability to subitize by immediately recognizing 5 objects in a line as 5 and then counting on 1 more resulting in a cardinality of 6.

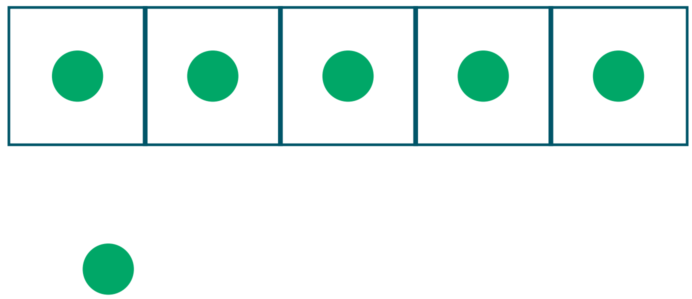

Figure 2. Five-frames help students develop their ability to recognize a group of 5.

HOW DO YOU FACILITATE AN UNDERSTANDING OF CARDINALITY?

The teaching methods and representations vary as cardinality develops. Research suggests that cardinality can be taught daily using routines, games, or visual models (Dougherty et al. 2010; Copley 2010). It takes time to develop this foundational idea and students need to see it many ways with a variety of materials. Many routines emphasize cardinality. Beginning with perceptual subitizing, *Toss and Count* is a routine that can begin early and progress developmentally as students state the cardinal number that answers the "how many" question.

Toss and Count

Toss a number of pom–poms or other counters. Count how many there are and respond with only one number. Repeat several times.

Lesson	1–2	3–1	5–2	7–4	
Number Range	1 to 3	1 to 7	6 to 10	6 to 10	

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Figure 3. An example of a routine that facilitates an understanding of cardinality.

To develop conceptual subitizing, encourage activities that require students to think about 5 in two parts. For example, students could use two-colored craft sticks to make stick pictures with 5 sticks, and then describe how many sticks of each color are in their pictures.

Figure 4. Two-colored stick pictures that represent the quantity 5.

Another example, the routine or partner game, *How Many Are Hiding?*, prompts students to think of 4 and 5 by having the teacher or another player cover some of the counters.

How Many Are Hiding?

Show five (or four) counters. Then hide some of them. Students guess how many are hiding.

Lesson	2–2	4–4	6–6	7–1	8–4
Number Range	1 to 5	0 to 5	1 to 5	0 to 5	1 to 5

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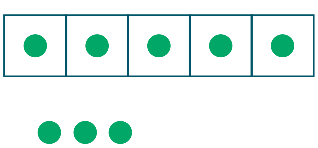
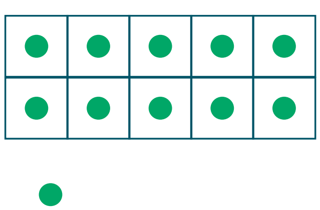
Figure 5. A routine that focuses on conceptual subitizing.

To extend the idea of subitizing for cardinalities of 6–10, five-frames and ten-frames can be used to help students subitize and count on if necessary to identify how many counters there are. The following pictures illustrate how students have expressed the cardinality of the sets using frames as representations.



"Five and 3 more"
"**5**...6, 7, 8
There are 8 dots."



"Ten and 1 more"
"**10**...11
There are 11 dots."

Visual models that help student see quantities within five-frame and ten-frame models can be part of lessons that emphasize counting and cardinality. These can be illustrated as students solve problems like this one: "Jackson's dog, Rex, has some balls on the red rug. Use counters and draw a picture on the empty dog bed to show how many balls Rex has. Tell how you know you are correct." Note the use of a five-frame to help students extend subitizing for 6–10.

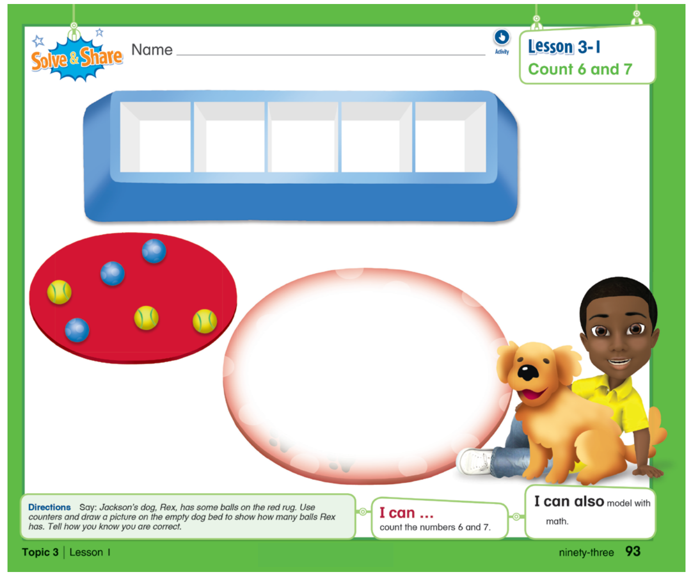


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Figure 6. A five-frame helps students extend subitizing for 6–10.

Estimation activities also provide opportunities for cardinality. A set of 10–30 objects can be displayed in a clear bag so that students can estimate the number of objects without touching the bag. Students estimate if there are MORE than 20 objects in the bag, EXACTLY 20 objects in the bag, or FEWER than 20 objects in the bag. Then the students who are counters of the day place all the objects on two ten-frames and report the answer as MORE than 20 (because there are leftover objects, which don't fit on the two ten-frames), EXACTLY 20 (because both ten-frames are filled with no objects left over), or FEWER than 20 (because the two ten-frames are not filled).

Finally, teachers must always emphasize the one-number answer to "how many" questions. Because my instruction has often been videotaped, I have had the opportunity to view my own teaching practices. When I ask students "how many" questions, I often just repeat the counting sequence rather than emphasize the cardinality of the set, which I should do instead. As teachers, we must intentionally end any and all "how many" questions by saying only one-number answers.

2. Object Counting

Accurate object counting requires using one-to-one correspondence to count objects with one count per object while keeping track of what has been counted and what hasn't been counted. To be accurate, the verbal number words (see the following section, Verbal Number Word List) must be coordinated with the procedure of one-to-one correspondence counting. For example, many state standards describe object counting as follows:

> When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

There are many counting concepts and procedures that students need to investigate, understand, and practice to master this foundational component.

- Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
- Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration.
- Understand that each successive number name refers to a quantity that is one larger.
- Given a number from 1-20, count out that many objects.

HOW DO YOUNG CHILDREN DEMONSTRATE THEIR UNDERSTANDING (OR THEIR PARTIAL UNDERSTANDING) OF OBJECT COUNTING?

I introduced a large cube to my group of kindergartners. I identified the faces of the cubes for students by rubbing my palm on the "bumpy" face and then rubbing my palm on the flat, smooth faces of the cube. I asked students to help me count how many faces there were on the cube. I then rotated the cube to a new face as they counted. I kept rotating the cube and they kept counting until someone said, "I think we counted that one already!" Indeed, we had and were now up to 13 faces, but I acted confused. When I asked them how we could keep track of our counting, several students suggested that I mark the sides in some way so we would know we had "counted that one already."

NOTE: I love to introduce the idea of "keeping track" in this way rather than planning it for them or telling them to mark the faces. Students remember to keep track when they see the need and discover a method in this way. It is just another example of the power of students' exploration and problem solving.

Prekindergartner Adrian put 5 Unifix® Cubes on the fingers of his right hand. He counted the cubes orally as I touched them. He said he had five cubes. When he switched 2 cubes to his left hand, leaving 3 on his right hand, I asked him how many cubes he had now. He said, "2 and 3," while he motioned with the corresponding hands. I asked him to count them again as I touched each cube. He counted in this way, "1, 2, 3," and then "1, 2." When I asked him to put all the cubes back on his right hand and then asked him how many he had now, he responded, "5 cubes." I said, "Isn't that funny? It was 5 and then it was 3 and 2 and then it was 5 again. I don't get it!" He shrugged and said, "Me too!" NOTE: Adrian did not see the same total when the cubes were decomposed into two groups as when they were together. In his mind, the number of cubes changed when they were separated. When he changed hands, he started counting with "1" again.

Kindergartners Jessica and Ryan were each playing with 9 counters. I asked them to put them in a line and count them. They did that well. They were able to tell me how many were in each line. I then asked them to place the same number of counters in a circle and count them again. This time, Ryan counted them correctly, remembering when to stop going around the circle. Jessica counted the 9 counters as 10, counting one of the counters twice. She simply didn't know when to stop counting as she went around the circle. When I asked her, "Why did that happen?" She thought for a few seconds and said, "I did it wrong." She counted again and this time stopped when she said 9 "cause that's how many I did before." As a last activity, I asked both children to place their counters in a plastic cup, shake it, and toss the counters randomly on the table. When I asked them to count them again, Ryan said, "I don't have to... I already know it is 9." Jessica wasn't sure and she counted them by picking up each counter and placing it back in the cup, correctly getting the answer of 9 counters. NOTE: Ryan understands that the arrangement of objects does not change the number count. Jessica appears to have an understanding of one-to-one correspondence; however, she is not sure about how to count the objects in different arrangements.

HOW DO YOU FACILITATE UNDERSTANDING AND ACCURACY OF OBJECT COUNTING?

Object counting requires an understanding of counting concepts, models of counting procedures, and practice coordinating the verbal number word list with one-to-one correspondence. Just like cardinality, object counting should be practiced daily using a variety of routines, games, and visual models. In addition, for kindergartners, the number of objects to be counted should increase gradually over time, beginning with groups of 5 or less and going up to 20 objects.

Problems that require accurate counting of objects provide excellent ways to help students develop an understanding of the counting principles.

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Figure 7. Examples of object-counting activities that require accurate counting of objects and help student develop their counting skills.

Representations of object counting that show the results of sorting can also help students develop an understanding of the importance of accurate counting. Kindergartners sorted the bug counters into two groups, one group contained the green bugs and the other group contained bugs that were not green.

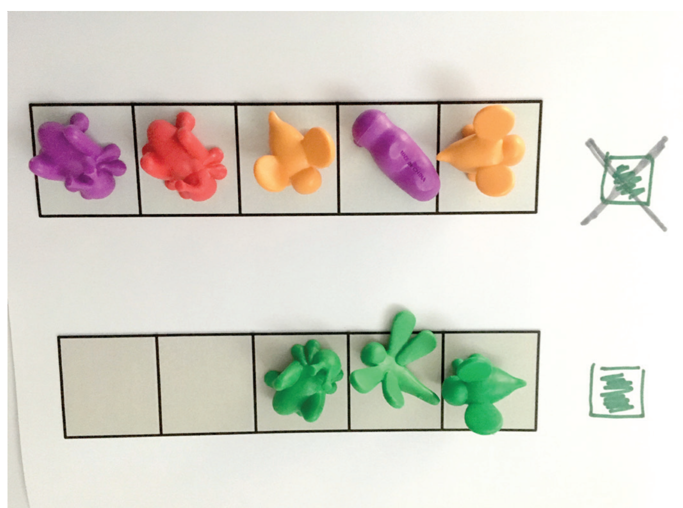

Figure 8. Representations of object counting that show sorting and data using five-frames.

Daily routines help students develop object-counting accuracy. For example, *Ooh! and Aah! Counting* integrates physical actions with oral counting 1–5, 1–10, and 1–15. *Counting Dots* provides a matching activity that involves counting dots on dominoes.

Ooh! and Aah! Counting

Students count using a pattern. They tap their legs as they count (1, 2, 3, 4, 5). They roll their hands and say *"Ooh!"* (because 5 is a special number!). Continue to count, tapping your legs each time (6, 7, 8, 9, 10). They roll their hands and say *"Ooh!"* (because groups of 5 are special!). Then students hold both hands palms out with all ten fingers extended and say *"Aah!"* (because groups of 10 are really important!). Count as high as you desire, saying *"Ooh!"* at every five count and extending fingers and saying *"Aah!"* at every ten count.

Lesson	1–4	3–5	8–2	9–2	
Number Range	1 to 5	1 to 10	6 to 10	1 to 15	

Counting Dots

Using double–nine dominoes (large or small), students match or identify the number of dots on one or two parts of a domino.

Lesson	9–4	9–7	14–2		
Number Range	1 to 20	1 to 20	1 to 20		

Source: ***enVision***® *Mathematics ©2020, Kindergarten (Charles et al., 2020).*

Figure 9. Two routines that emphasize object-counting accuracy.

3. Verbal Number Word List

An essential part of accurate counting is saying the number words in order verbally without leaving any numbers out of the sequence. Of course, if there are any verbal errors, the cardinality of the set will be incorrect. The verbal number word list must be memorized. State standards require children to count to 100 verbally (1–20 object counting and 21–100 as only a verbal sequence). They will need to **count to 100 and by tens.** In addition, they must be able **to count forward beginning from a given number within the known sequence.**

Children best memorize words and numbers by identifying patterns and generalizable information. Unfortunately, in English (and Spanish), there are few language patterns within the first 20 numbers. Research suggests that the verbal number word list be memorized in the following steps (The Early Math Collaborative 2014; Dougherty et al. 2010; Clements and Sarama 2009).

1. Memorize the beginning sequence (1–5); (1–10).
2. Highlight the few verbal patterns in the teen numbers (11–19) ("fourteen," "sixteen," "seventeen," "eighteen," and "nineteen") AND the numbers that don't follow the verbal pattern ("eleven," "twelve," "thirteen," and "fifteen"). Also note that unlike the numbers 20–99, the ones digit in the teen number is said first and then the "teen," or the tens number, is expressed.
3. Highlight the verbal patterns for numbers 20–99. The overall pattern is that the number words begin with a decade number and end with 1–9.
4. Focus on the (almost) verbal pattern of decade numbers, specifically, twenTY, thirTY, FORTY, fifTY, SIXTY, SEVENTY, EIGHTY, and NINETY.

HOW DO YOUNG CHILDREN DEMONSTRATE THEIR ACCURATE USE OF THE VERBAL NUMBER WORD LIST?

One of my roles as a mathematics coach involved assessing the oral counting of 4- and 5-year-olds. As part of the initial assessment, I would simply ask, "Can you count?" and children would begin verbally reciting the counting sequence. The three responses below illustrate some of the most interesting responses:

> "One, three, four, five, six, seven, eight, nine, ten, YELLOW (or maybe it is ELEVEN). It is hard to remember!"
>
> "One, two, three, four, you are going to be in BIG trouble!"
>
> "One, two, three, four, five, six, seven, eight, nine, ten, teen, teen, teen, teen, teen, teen, teen, one hundred!"

While these responses are humorous, they demonstrate the difficulty that occurs with the English counting words. The words for 11 and 12 do not follow any type of pattern, and the non-pattern forms of the rest of the teen numbers are difficult as well.

Kindergartner Tucker was able to verbally count accurately from 1 to 60 as long as he could begin at 1 and keep his song-type recitation. When I asked him what number was "1 more than" another number or "next after" another number, he would begin again at 1, try to get to the number I mentioned, and then tell me the next number. That procedure worked pretty well until we reached numbers greater than 20 and Tucker couldn't remember where to stop. For example, 35 came next after 31 and 55 came next *after 49. When I asked him what number was 1 more than 100 or next after 100, he gave the typical answer for a 5-year- old… 200!* NOTE: The idea of 1 more than a number is an important concept for young children, especially when learning to add by counting up. Initially, when I questioned Tucker, I said, “What number is next?” rather than “What’s 1 more?” Right or wrong, he easily gave a number when I asked the first question. When I asked the second question, he appeared to be confused. When a visual counting model is introduced at the same time as the oral counting model (see the following section, Reading and Writing Numbers), Tucker and students like him will be better able to understand this concept.

HOW DO YOU FACILITATE THE MEMORIZATION OF THE VERBAL NUMBER WORD LIST?

To facilitate the memorization of the number word sequence, children can match the visual patterns (which are easily seen for all numerals) with the verbal words. The problem pictured below is designed for partners and is an example of how memorization of the number sequence can be practiced. One partner points to the numbers in each row beginning with 1 while the other partner counts verbally. They switch roles after each row. After partners have gone through the entire grid, one partner covers some of the numbers with counters. The other partner finds the hidden numbers and verbally states them. After they have played the “hidden numbers” game three times, partners color the numbers that are the hardest to remember.


Solve & Share Name

Lesson 11-2
Count by Ones and by Tens to 50

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Directions Say: *Work with a partner. Count forward from 1 to 50. One partner points to each number in the first row while the other partner counts aloud each number. Change jobs for every row. Watch students go through all 5 rows.* Say: *Now, one partner will cover some numbers on the board using counters. The other partner will name the hidden numbers. Play 3 times, and then color the numbers that are the hardest to remember.*

I can … use patterns to count to 50.

I can also make sense of problems.

Topic 11 | Lesson 2 four hundred thirty-seven 437

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Figure 10. A problem for kindergartners: “Find and say the hidden numbers.”

Counting verbally by 10s can and should be visualized with different models. In each of the representations below, students make groups of 10 and then count by 10s to reach 100.

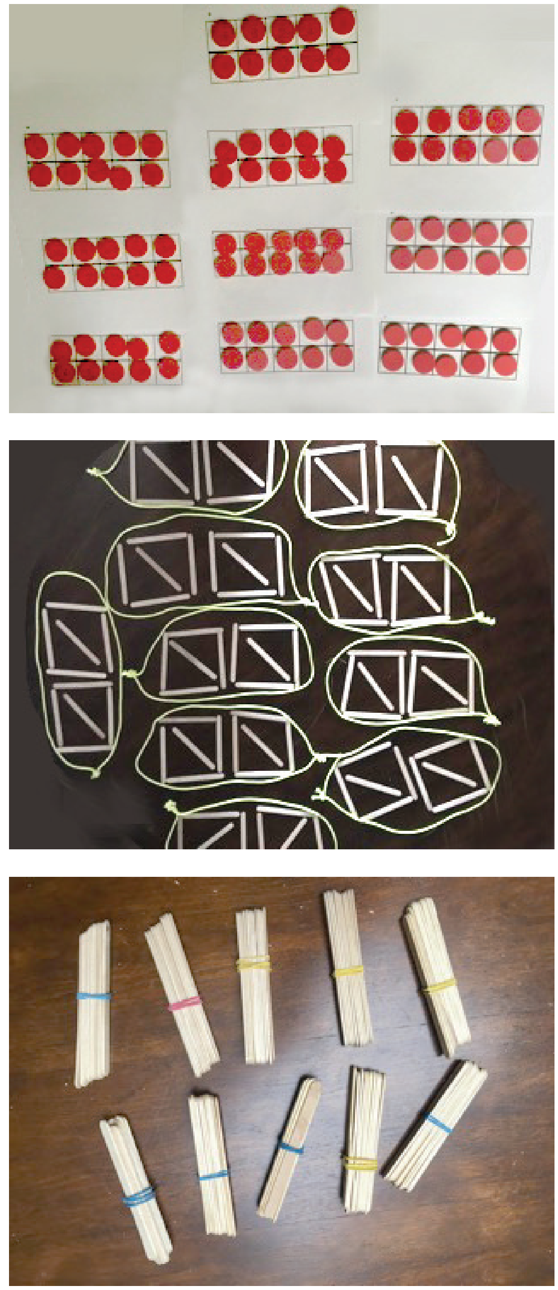

Figure 11. Visual models for groups of 10 that are used to verbally count by 10s to 100.

The counting sequence must be practiced throughout the year; it should not be introduced and then ignored, nor should it be the only focus of kindergarten mathematics instruction every day. Effective practice can best occur during short routines that are intentionally planned to include more numbers in students' memorized number sequence. For example, *Silly Sam* provides a wonderful opportunity for children to identify errors in the counting sequence. *Exercise by Decade* can help students memorize the numbers within the decade as they count using physical movements.

Silly Sam

Silly Sam is a puppet. He always tries to count objects, but ends up getting the sequence incorrect, leaving out numbers. Children correct him by telling what numbers he missed.

Lesson	1–10	3–4	5–1	7–3
Number Range	1 to 5	1 to 9	6 to 10	6 to 10

Exercise by Decade

Students select a movement for the numbers in each decade (e.g., jumping jacks for the numbers 40–49). They count each decade doing that specific movement. Typically, they select only one or two decades to exercise and practice. Sometimes, they may count 1–100, exercising during each decade as they count.

Lesson	11–4	12–2	13–2	13–6
Number Range	1 to 100	1 to 100	1 to 100	1 to 100

*Source: **enVision**® Mathematics ©2020, Kindergarten (Charles et al., 2020).*

Figure 12. Routines that provide practice for repeating the number sequence 1–100.

4. Reading and Writing Numbers

At the kindergarten level, students read and write numerals from 0 to 20 and represent a number of objects with a numeral 0–20. Further, kindergarteners are expected to verbally count from 1 to 100. Because there is a strong visual pattern in the number sequence and not as many patterns in the oral verbal sequence, I have found that it is very helpful for students to see the numbers 1–100 as they count. On a daily basis in the classroom, we select a beginning number for our oral counting practice and designate a child to be the "pointer." That child moves the pointer along the number line or on a hundred chart as we say the numerals. Students identify the numerals that they say incorrectly or the ones that do not follow the patterns. They highlight and practice those numerals frequently as partners, small groups, or with the whole class.

HOW DO YOUNG CHILDREN DEMONSTRATE THEIR SKILLS IN READING AND WRITING NUMBERS?

Several years ago, I tried an experiment in my prekindergarten classes. I hung a completed hundred chart on one wall in each classroom. In all cases, children were curious about the numerals on the chart and were finding "their numbers" (usually meaning their ages) on the chart. When I identified my age and found it on the chart, students were amazed and one child informed me, "You are going to pass!" When I inquired what he meant, he replied, "You are almost at the end!" NOTE: This humorous response prompted me to think about another benefit of the hundred-chart display. As children observe the numbers on the hundred chart, they are able to see that the numbers become greater as they count forward.

HOW DO YOU FACILITATE THE READING AND WRITING OF NUMBERS?

Modeling: The most common method is modeling both the reading and the writing of numbers using real-life situations. Examples include recording the lunch account, conducting a number-search field trip, and writing bus numbers on the going-home chart.

Displays: Just as letters are part of every kindergarten classroom, numbers are an equally important part of the kindergarten classroom environment. One way to display numbers is to write numbers on a number line that shows the entire counting sequence from 1 to 100. Also, students should be given opportunities to trace numbers and encouraged to label their drawings with numbers.

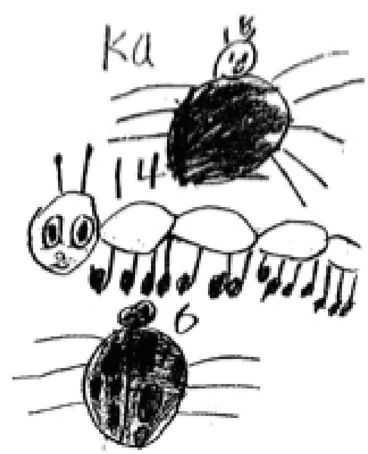

Figure 13. A kindergartner's drawing of mini-beasts with the number of legs labeled.

While kindergarten students are not expected to read and write numbers 21–100, they benefit from seeing visual models as they count. For example, the visual models shown below support comprehension as students count by 10s to 100.

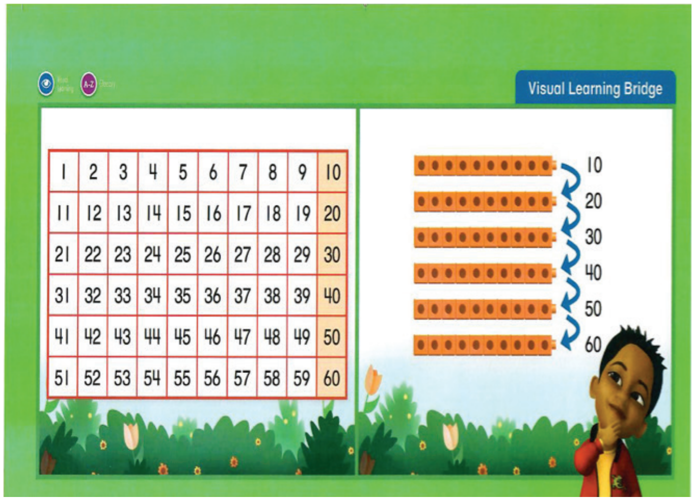

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Figure 14. Visual models help kindergarten students develop understanding of the counting sequence.

Practice: Incorporate daily routines that involve reading numbers from 1 to 20, tracing and writing numbers from 1 to 20, and oral counting from 1 to 100 (supported by visual displays).

One of the favorite routines for kindergartners is to count all the words they can read. To help them see the number of words in numerical form, I make an empty hundred chart entitled "100 Words I Can Read." As the year progresses, each square on the chart is filled with a word that students can read. Normally, the chart begins with environmental print, continues with relevant nouns, and ends with basic sight words. The decade numbers are labeled at the end of each row to help students see the written numerals as they count the rows by 10s.

5. Cardinal Counting

Cardinal counting is the term used to describe the coordination between cardinality and counting, which is often thought of as the transition to place-value concepts. For kindergartners, it is the idea that 10 ones equals 1 ten and that the teen numbers can be thought of as all ones or as 1 ten and some ones. These concepts are addressed in most state standards. For example, consider these Kindergarten standards:

> For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
>
> Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

HOW DO YOUNG CHILDREN DEMONSTRATE CARDINAL COUNTING?

In most cases, young children begin to understand the importance of the number 10 during the kindergarten year as they experience the counting sequence on a routine basis.

"One, two, three, four, five, six, seven, eight, nine, ten, ten-one, ten-two, ten- three, ten-four, ten-five, ten-six, ten-seven, ten-eight, ten-nine, two ten." was said by a kindergartner as a "new way of counting to 20." NOTE: She knew the "old" way (eleven, twelve, thirteen, and so on) because she had memorized it. However, her new way showed she understood the concept.

When asked to complete a sentence, "I like 10 because...," one child responded, "...we have 10 fingers and it makes it easy to count all the cubes in the basket!" NOTE: I always ask children in the classroom to clean up our materials by counting out objects into groups of 10, count ten groups of 10, and place 100 total cubes in a container.

HOW DO YOU FACILITATE THE UNDERSTANDING OF THE COORDINATION BETWEEN CARDINALITY AND COUNTING?

Just as in the previous four principles, daily routines are the most effective in developing an understanding of cardinal counting. Routines that focus on the decade numbers and forming groups of 10 to count specifically address the coordination between cardinality and counting. For example, both oral counting by 10s and visualizing groups of 10 are integrated in the routine, *Hiding Tens*. In a similar way, *100...More or Less?* is an activity that requires an understanding that ten groups of 10 is the same as 100 and the skill to make groups of 10 to count by 10s.

Hiding Tens

Display ten rows of ten cubes so students can see the individual rows. Students close their eyes while one student covers some of the rows. Students count the rows of ten that are uncovered and guess how many rows of ten are covered.

Lesson	11–3	12–5	13–3	14–3	
Number Range	1 to 100	1 to 100	1 to 100	1 to 100	

100 ... More or Less?

Display a plastic bag of 75–125 objects where students can see it but not touch it. Students vote if they think there are a) fewer than 100 objects in the bag, b) more than 100 objects in the bag, or c) exactly 100 objects in the bag. Then, the counter of the day empties the bag, places the objects on ten–frames, and reports the total number of objects.

Lesson	11–5	12–7	13–4	14–6	
Number Range	1 to 100	1 to 100	1 to 100	1 to 100	

Source: ***enVision***® *Mathematics ©2020, Kindergarten (Charles et al., 2020).*

Figure 15. Examples of routines that connect cardinality and counting by focusing on forming groups of 10 and counting to 100 by 10s.

5. Conclusion

Without a doubt, counting is foundational and essential to understanding in mathematics. Counting is more than memorization, more than one-to-one correspondence, and more than procedural skills. In this paper, we have examined five principles for cardinality and counting. These five principles include: **(1)** cardinality (including subitizing), **(2)** object counting, **(3)** the verbal number word list, **(4)** reading and writing numbers, and **(5)** cardinal counting. If these principles are developed through routines and visual representations that focus on them, kindergarten students will be prepared for the mathematics of first grade.

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