



Video

## TOPIC 12

# Mathematical Modeling in 3 Acts: Earth Watch

## Lesson Overview

FOCUS

### Objective

Students will be able to:

- ✓ Use mathematical modeling to represent a problem situation and to propose a solution.
- ✓ Test and verify the appropriateness of their math models.
- ✓ Explain why the results from their mathematical models might not align exactly with the problem situation.

### Essential Understanding

Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly.

COHERENCE

Earlier in this topic, students:

- Solved problems using arcs, sectors, and tangent lines.

In this lesson, students:

- Develop a mathematical model to represent and propose a solution to a problem situation involving circles.

Later in this course, students will:

- Refine their mathematical modeling skills using angles and measurements in circles.

RIGOR

This mathematical modeling lesson focuses on application of both math content and math practices and processes.

- Students draw on their understanding of concepts related to measurements in circles to develop a representative model.
- Students apply their mathematical model to test and validate its applicability to similar problem situations.

MATHEMATICAL  
MODELING  
IN 3 ACTS  
  
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Video

**Earth Watch**

Scientists estimate that there are currently about 3,000 operational man-made satellites orbiting Earth. These satellites serve different purposes, from communication to navigation and global positioning. Some are weather satellites that collect environmental information.

The International Space Station is the largest man-made satellite that orbits Earth. It serves as a space environment research facility, and it also offers amazing views of Earth. Think about this during the Mathematical Modeling in 3 Acts lesson.

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**ACT 1** Identify the Problem

1. What is the first question that comes to mind after watching the video?
2. Write down the main question you will answer about what you saw in the video.
3. Make an initial conjecture that answers this main question.
4. Explain how you arrived at your conjecture.
5. What information will be useful to know to answer the main question? How can you get it? How will you use that information?

**ACT 2** Develop a Model

6. Use the math that you have learned in this Topic to refine your conjecture.

**ACT 3** Interpret the Results

7. Did your refined conjecture match the actual answer exactly? If not, what might explain the difference?

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## Mathematics Overview

### The Importance of Modeling

Rather than presenting a problem with all the necessary information, diagrams, and graphs in advance, this task also asks students to participate in the formulation of a problem.

Students decide what questions are interesting, what quantities are important, and how they could access information. They choose which math concepts are relevant to the task, using approximations or assumptions in the solution when necessary.

Finally, the task includes an actual, real-world answer. Students discuss possible sources of error inherent in using math to model real-world situations. Then they evaluate the usefulness of their models in the situation, improving them if necessary.

### Mathematical Practices

#### Model with Mathematics

To solve the problem presented, students identify variables and the relationship among them, develop a model that represents the situation, and use the model to propose a solution. Students interpret their solutions and propose explanations for why their answer may not match the real-world answer.

#### Other Practices

Students also engage in sense-making, perseverance, and abstract and quantitative reasoning as they complete the task. In testing their models, students look for patterns in their models and use that structure to find a solution.



Video

## TOPIC 12 Mathematical Modeling in 3 Acts

### Earth Watch

In this mathematical modeling task, students will explore and apply concepts related to theorems about circles. Students will analyze a situation in which two satellites orbit Earth. They will be tasked with determining which satellite has the better view of Earth. To do so, they apply concepts that they study in Topic 10.

#### ACT 1 The Hook

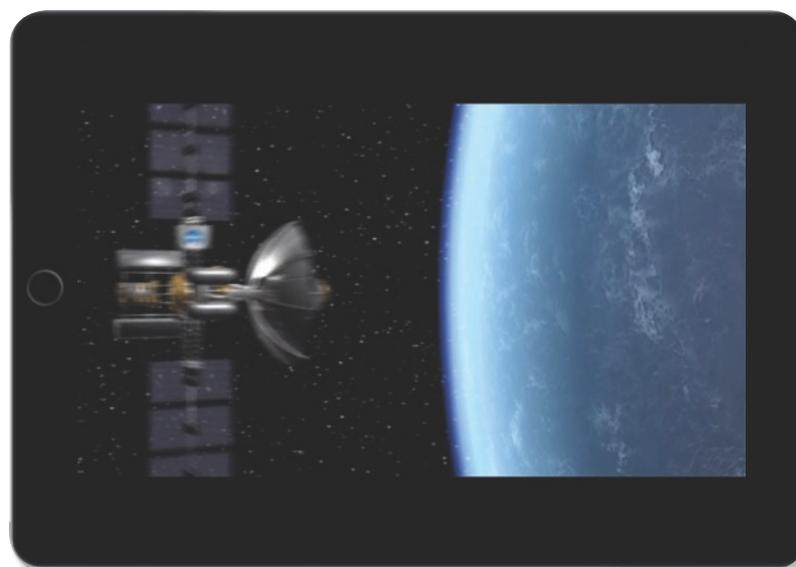
**Play the video.** The video shows two satellites at different altitudes in orbit above Earth. After the question brainstorming, present to students the Main Question they will be tasked with answering. Remind students to write down their questions and conjectures.

#### MAIN QUESTION

Which satellite has a better view of Earth?

#### INTERESTING MOMENTS WITH STUDENTS

Discuss with students what the phrase *better view* means in this situation. Lead them to define the phrase in terms of the arc of Earth that can be seen from the satellite.



#### ACT 2 Modeling With Math

**Think about the task.** Ask students to speculate how they could determine which satellite can see more of Earth. Then have them think about what information they would need to find the answer.

**Reveal the information.** Once students ask, use the video to reveal the information about the satellites and Earth. Then have students use the tool to make constructions and measurements they think will help them solve the problem.

**What's the connection?** Give students time to struggle as they think about how their ideas are connected to what they learned in this topic. How can they answer the Main Question with and without a scale drawing?

#### INTERESTING MOMENTS WITH STUDENTS

Students may assume that three times the altitude means three times the angle. Refrain from pointing out this misconception right away. They will eventually realize that they need to account for the radius of Earth, which makes the distance from the center of Earth four times as much. Even then, there isn't a linear relationship between the altitude of a satellite and its viewable arc.

#### Necessary Information

##### Satellite altitudes

near satellite: 6,400 km above Earth

far satellite: 19,200 km above Earth

##### Radius of Earth

6,400 km



## TOPIC 12 Mathematical Modeling in 3 Acts

### ACT 3 The Solution

**Play the video.** The final video reveals the measure of the arc of Earth that each satellite can view. Offer praise to the students whose conjectures are closest to the actual answer.

#### MAIN QUESTION ANSWER

The near satellite has a  $120^\circ$  view of Earth, and the far satellite has a  $151^\circ$  view of Earth. The far satellite has a better view.

**Do the “post-game” analysis.** Now is the point at which students can share other variables that might affect what “better view” means in this situation. For example, the farther satellite has a larger angle, but it would likely require more expensive equipment to take the same quality images from three times the distance.

#### ONE POSSIBLE SOLUTION

In the diagram (not to scale),  $A$  represents the near satellite and  $D$  represents the far satellite.  $\overline{AB}$  and  $\overline{AC}$  are tangents to Earth through point  $A$ , and  $\overline{DF}$  and  $\overline{DG}$  are tangents through point  $D$ . Using Theorem 12-1, we know  $\angle B$ ,  $\angle C$ ,  $\angle F$ , and  $\angle G$  are right angles.

Use right triangle trigonometry to find  $m\angle CEA$  and  $m\angle FED$ .

$$\cos(m\angle CEA) = \frac{6400}{6400 + 6400} \qquad \cos(m\angle FED) = \frac{6400}{6400 + 19,200}$$

$$m\angle CEA = \cos^{-1}\left(\frac{1}{2}\right) \qquad m\angle FED = \cos^{-1}\left(\frac{1}{4}\right)$$

$$m\angle CEA = 60^\circ \qquad m\angle FED \approx 75.5^\circ$$

By Theorem 12-3, we know that  $\overline{AC} \cong \overline{AB}$  and  $\overline{DF} \cong \overline{DG}$ . Because all radii of a circle are congruent,  $\triangle ACE \cong \triangle ABE$  and  $\triangle DFE \cong \triangle DGE$  by the Side-Angle-Side postulate. So  $m\angle BEA = m\angle CEA$  and  $m\angle GED = m\angle FED$ . Therefore,  $m\angle CEB = 120^\circ$  and  $m\angle FEG = 151^\circ$ .

Since both  $\angle CEB$  and  $\angle FEG$  are central angles, the arcs of Earth viewed from the satellites are also  $120^\circ$  and  $151^\circ$ , respectively.

#### INTERESTING MOMENTS WITH STUDENTS

**Each arc corresponds to surface area of Earth.** Though the videos show a 2-D representation of Earth, the satellite's view is of a circular portion of Earth. Give students the opportunity to think about how the surface area increases as the measure of the corresponding arc increases.

#### SEQUEL

As students finish, ask them to find the greatest arc of Earth that a satellite could view. How far from Earth would the satellite have to orbit? [Answer: As you get farther away, you would approach a  $180^\circ$  view of Earth. Even infinitely far away, you will never see exactly half of Earth at one time.]

