

3-Act Mathematical Modeling: Authentic Engagement with Mathematical Ideas

By Zachary Champagne and Jennifer Suh

Part 1: What Is a 3-Act Math Modeling lesson?

Overview and Structure

3-Act Math Modeling lessons are built upon this foundational idea: *Students are more engaged in mathematics when they are authentically invested in the task.* As will be detailed in this paper, this investment is far beyond that which students generally experience with a traditional “real-world” task.

The basic structure of a 3-Act Math lesson is based on storytelling. Books and movies often tell their story in three parts: conflict is introduced, characters look for clues and resources, conflict is resolved. Mathematics educators (Meyer, 2011) have noticed that this framework for storytelling maps nicely onto high quality math tasks. These lessons begin with “conflict”: an intriguing image or video that is intended to pique the student’s interest. From there students are encouraged to pursue questions that they have based on the video, consider what information they need to find the answers to those questions, and finally (“conflict resolved”) use mathematics to answer the question.

What Happens in Act 1?

In Act 1 of these lessons, the teacher presents a striking visual (image or video vignette) that is intriguing and engaging—intended to draw students into



Figure 1. A girl thinks about the ways to order pizza.

the problem. Important to Act 1 is simplicity—but simplicity that requires students to want to know and to do more. Dan Meyer (2011) says, “Your first act should impose as few demands on the students as possible—either of language or of math. It should ask for little and offer a lot.”



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Most of the 3-Act Math lessons in *enVision® Mathematics* have video vignettes for Act 1. As an example, consider the following screen shots from a Grade 7, 3-Act Math lesson.

Students watch a girl prepare to order pizza. She considers ordering four small pizzas, but ultimately asks if she can order a very large pizza.



Figure 2. The girl requests a giant pizza.

After students watch the video (or view an image) in Act 1, the class brainstorms a multitude of questions that the class community has. Students are encouraged to discuss all the questions from their classmates, and through discussion as a group (including the teacher), the class settles on a main question to pursue. In this example, the *main questions* would be, “How much will the giant pizza cost?” and “Which option is the better deal?”

In another critical component of Act 1, students estimate, or predict, the answer to the main question. This estimate is much more than just a guess. They closely watch the video again and discuss what would be a reasonable answer to the main question. In this example, students use what they know about their own experience with pizza costs to determine a reasonable estimate for the cost of the giant pizza.

What Happens in Act 2?

In Act 2, students begin to consider what information they would need to be able to answer the main question. The teacher’s role is to provide that information. The key here is that students are now looking for the information they need to solve the problem—one of the pillars of a powerful problem-solving task.

In this example, the teacher presents these two follow-up images to offer the information students need.

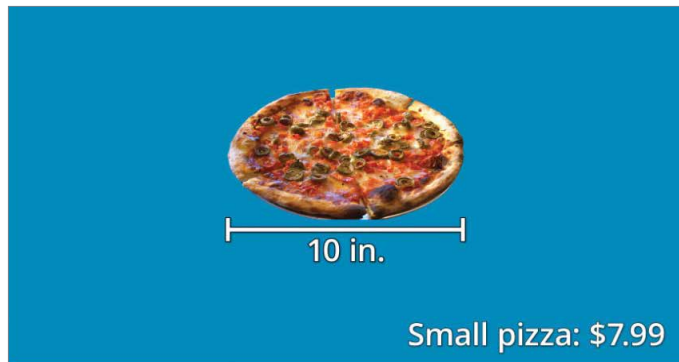


Figure 3

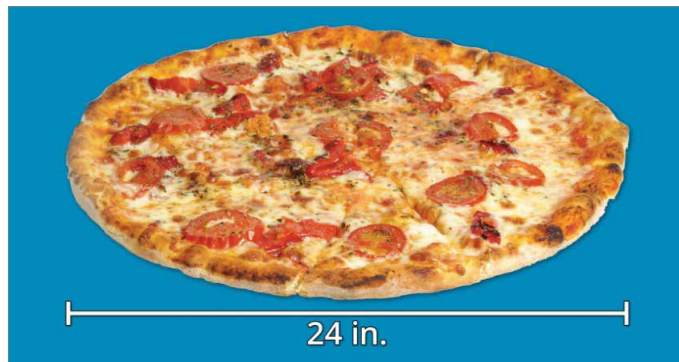


Figure 4. Information about pizza sizes and cost is provided.

What Happens in Act 3?

In Act 3, students get the payoff for their hard work: *The answer to the main question is revealed.* The word “revealed” is important here. Remember, students have a vested interest in this problem. They deserve a resolution that is powerful and much bigger than “The pizza is \$49.99.” In this lesson, the video reveals how the manager of the pizzeria quickly estimates the price of the pizza.



Figure 5. The pizzeria manager and worker estimate the cost of the pizza.

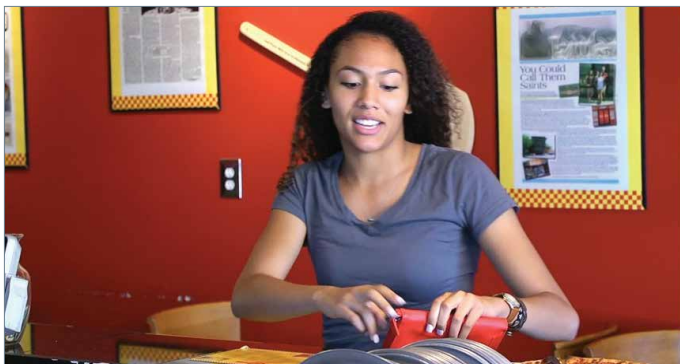


Figure 6. Sharing the price with the customer.

The payoff will be noticeable. Once you do a 3-Act Math lesson with your students, you'll find that in Act 3, there is almost always a cheer. Students are invariably excited to see an authentic reveal of the solution to a problem that they chose to solve, and see it solved in a way that made sense to them.

Oftentimes, after Act 3, “sequels are provided for students to explore. These sequels push further questions and ideas that are related to the mathematics involved in the task.”

Important Note: In this particular task, students are provided in Act 2 with some of the exact information that is necessary to determine the solution to the main question. This solution has some ambiguity to it because the manager made an estimate based on a ratio, but then rounded it up a bit “for the trouble.” The information given isn't exactly clear, and estimations or approximations are necessary. This case is an example of when the solution in Act 3 brings somewhat of a surprise or twist to the lesson. This is intentional, and helps students understand the interaction between mathematics, modeling, and the real world.

Part 2: Why 3-Act Math Modeling Lessons?

3-Act Math lessons engage, involve, and challenge students at all levels. A well-constructed 3-Act Math lesson allows each and every student in your classroom, on some level, to access the mathematical ideas inherent in the work. It also provides paths for students to pursue the question(s) at higher levels. For example, in the task above, students can pursue a variety of follow-up questions, including but not limited to these:

- How can we express the relationship between the size of the pizza and the cost?
- Should the cost be based on circumference or area?
- Is there another way to solve this problem other than the option I have chosen?
- How many people do these options feed?

A second, yet equally important, component of these tasks is that students have the chance to ask authentic questions. When teachers ask, “*What questions do you have?*,” they are genuinely interested in hearing how students respond, because there are no expected answers; no one knows what questions the students might have. As teachers show that they are invested in the students' expressed ideas, the students themselves become authentic learners; both sides are authentically engaged in the conversation about the situation.

The estimation skills that students acquire through 3-Act Math lessons are remarkable. As students have continued conversations around estimates that are too high and estimates that are too low, they become better estimators as they use and think carefully about the information that they are given in Act 1. They learn to do less guessing and to think about the situation and various factors that may influence the solutions.

When students engage with 3-Act Math lessons, they are building agency and engaging in tasks where they have choice and input on what ideas are pursued. The impact of choice and self-determination on learning is well documented in the research and literature. In fact, providing students with choice and/or more autonomy has been shown to:

- Increase attendance and increase scores on a national test of basic skills than those in conventional classrooms. (de Charms, 1972)
- Help students develop more sophisticated reasoning skills without falling behind on basic conceptual tasks. (Cobb, et al. 1991; Yackel, Cobb, and Wood, 1991)

Part 3: 3-Act Math Modeling lessons and the Mathematical Practices and the Progression to Mathematical Modeling

These 3-Act Math lessons can support teachers and students in developing the mathematical practices as presented by the Common Core State Standards. One of the most significant mathematical practices supported by 3-Act Math lesson is the *Standards for Mathematical Practice MP4: Model with mathematics*. The video vignettes provide students a way to visualize the problem situation and relate the problems to real-life scenarios. Through engagement with each 3-Act Math lesson, students see that mathematics is used “to solve problems arising in everyday life, society, and the workplace” (CCSSM, 2010). In middle school, this may start with students writing an equation to describe a situation, then move towards the modeling process—making assumptions, building a solution, then revising

their thinking, by reflecting on the real-world situation and improving on their mathematical solution or model. This leads in a seamless progression to more formalized mathematical modeling tasks.

As students pose the problem in Act I, they are working to *make sense of problems and persevere in solving them* (MP1). As they talk about the task, students make sense of the problem by asking questions, promoting inquiry-based thinking. In Act 1 of a sixth grade 3-Act Math lesson featured below, students watch a video showing a teen packing a suitcase of tennis shoes for a trip. After prompting students to generate their questions, the teacher could pose one of their questions to engage the class, such as “How many pairs of shoes fit in his suitcase?” Looking at the photo, students will have to make some assumptions. They may have to consider the airline weight limit for the suitcase, the weight of the luggage, as well as the weight of the shoes.



Source: enVision® Mathematics ©2021, (Charles et al., 2021)

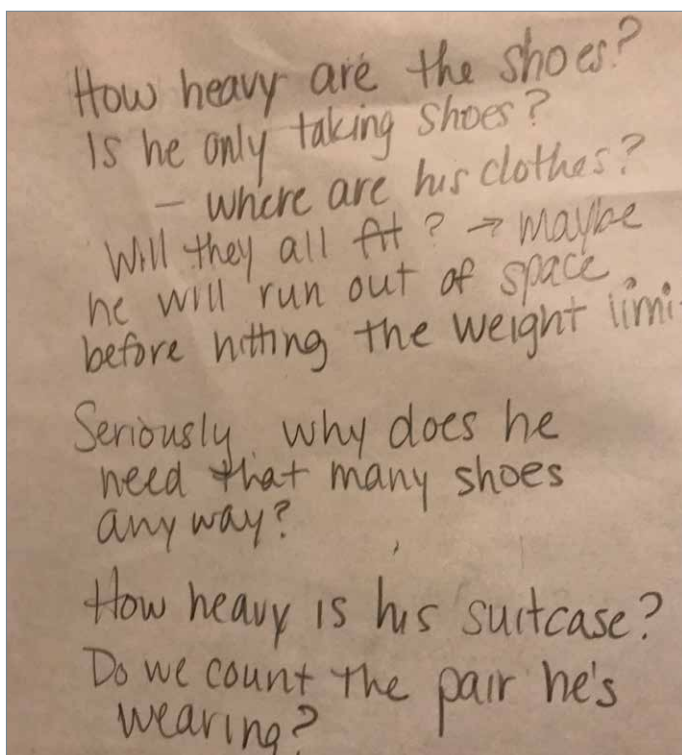
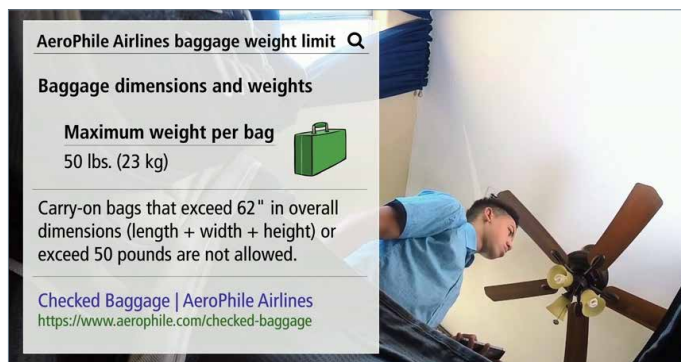


Figure 8. In Act 1, viewing an image generates questions.

In Act 2, teachers can focus on having students *reason abstractly and quantitatively* (MP2) as they make assumptions and consider what information they would need to answer the main question. Students make educated estimates that allow teachers to assess their number sense and quantitative reasoning skills.



Source: enVision® Mathematics ©2021, (Charles et al., 2021)

Figure 9. In Act 2, students are provided data and/or more information to mathematize the problem.

Based on the assumptions, students will determine pathways to solve the problem. For example, they may write a number sentence or build mathematical equations. The teacher may prompt students to *attend to precision* (MP6) as they give carefully formulated explanations to each other. A student thinking aloud might sound like this: “I am going to assume that this is an accurate snapshot of the weights of the shoes. This means I can average the weights and estimate that each pair of shoes weighs about 2.3 lbs. If p = the number of shoes, then $2.3p + 6.2 \leq 50$ so $p \leq 19.04$. He can pack 19 pairs of shoes.”

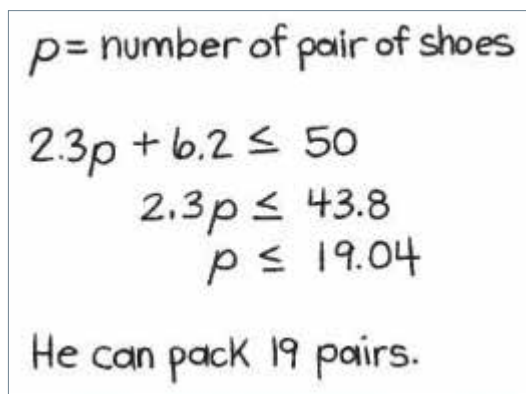


Figure 10. An example of a solution derived through a 2-step inequality.

In this example, students would also *look for and make use of structure* (MP7) in their computational work. For example, some students might use multiplication to solve the problem, while others might use repeated addition. As students

discuss and *critique the reasoning of others* (MP3), they may notice that there are patterns in their thinking. They may notice that some of them are using an inequality to represent the relationships in the problem, as shown above in Figure 10.

While comparing methods and considering the many different configurations, students will notice that the variables important to this problem are considering how much each pair of shoes might weigh, the airline's weight limit, and the weight of the suitcase. Students may use algebraic structures, methods, and patterns as they *look for and express regularity in repeated reasoning* (MP8).

Students draw on their understanding of equality and inequality concepts to develop a representative model. This use of tools supports *using appropriate tools strategically* (MP5) to represent their thinking.

Researchers have emphasized that in algebra problem-solving situations, it is not merely getting the answer that counts; just as important is emphasizing how these situations are about relationships among solving methods, and finding ways to represent these relationships and methods with equations that are as generalizable as possible (Kieren, 2007; Boaler and Humphreys, 2005).

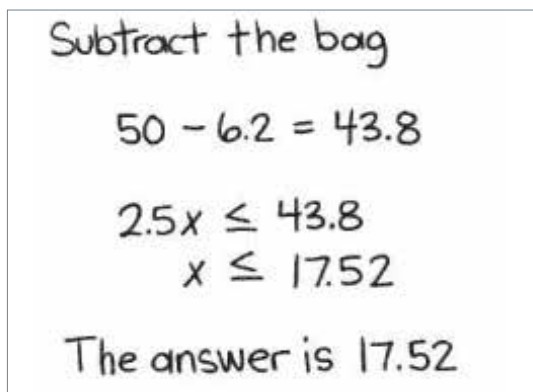


Figure 11. Visual representation of the number of shoes using algebraic thinking.

In other words, the algebraic inequality would allow for students to create a generalizable model. Another way to represent a generalizable model for calculating total number of shoes is: $2.5x \leq 50 - 6.2$, where x represents the number of shoes; 50 is the baggage weight limit, and 6.2 is the weight of the suitcase. Although some students may be at a more concrete stage with 2-step inequalities, others might verbalize, “I will subtract the weight of the suitcase and then divide the remainder by the heaviest pair of shoes. $43.8/2.5 = x$ so $x = 17.52$ ”

The most exciting part is providing students with the opportunity to engage in a healthy mathematical argumentation. Based on different assumptions, students may have different answers. However, an important aspect of these tasks is that students are given the opportunity to *construct viable arguments and critique the reasoning of others* (MP3). In so doing, they learn to value different perspectives and appreciate the variety of ways people find to solve problems.

When more information is revealed in the final Act, in a form of a photo or a video vignette, students are given an opportunity to compare their assumptions and the reasonableness of their solutions. In a culminating image of this 3-Act Math lesson, students get to see both the weight and shoe count increase with the addition of each pair of shoes. This is where they may revise their thinking as they realize, “Oh, there are a lot more lighter weight pairs of shoes than I expected.” (e.g., $50 - 6.2 = 43.8$, $2.5x \leq 43.8$, $x \leq 17.52$).

Conclusion

An overarching goal of including 3-Act Math Lessons in classrooms is to engage students in being both problem posers and problem solvers as they determine solutions to authentic questions. They are put in the driver's seat as they both pose the questions and determine the solutions to those problems. This sets our students up for success as they become people who can take problems from the real world and mathematize those situations, making wise decisions using the most important tool, mathematics! Early introduction to this idea of modeling allows for a natural progression from middle to high school standards for mathematical modeling.



Source: enVision® Mathematics © 2021, (Charles et al., 2021)

Figure 12. The final reveal allows students to compare their assumptions and the reasonableness of their solutions.

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