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Research into Practice:
MATHEMATICS

Language Support in Mathematics

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Understanding the Language of Schooling

One of the challenges faced by all students involves learning the specific vocabulary, grammatical structures, and conventions of use characteristic of different academic subjects. Each content area contains technical vocabulary that is largely specific to that discipline (e.g., in math this includes words like *addend, decimal, denominator, numerator, quadrilateral,* and so on) but there are also many general academic vocabulary words that are common across content areas. For example, mathematical problems typically involve words such as *explain, describe, demonstrate, identify,* and so on, which derive from Latin and Greek sources, and which also appear across the academic disciplines. Beyond vocabulary, the language structure of mathematical problems can be challenging for students. Some mathematical problems require students to understand relationships that are expressed by means of language structures that are very different from the language we use in everyday conversation.

The language demands of mathematics entail significant challenges for students who are struggling to acquire grade-level reading and writing skills and for English language learners (ELLs) who are in the process of learning English and catching up academically. ELLs may be relatively fluent in spoken English after one or two years of exposure, but considerable research has demonstrated that it typically takes at least 5 years, on average, for students to reach grade expectations in their command of academic English. This longer catch-up period for academic language, as compared to everyday conversational language, is caused both by the complexity of academic language and the fact that native speakers of English are not standing still waiting for ELLs to bridge the gap. Every year, native speakers increase their reading and writing skills as well as their knowledge of vocabulary and grammatical structures. Thus, ELLs must “run faster” to catch up to a moving target. The major differences between conversational and school language are summarized in Table 1.

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Conversational fluency	School language proficiency
The ability to carry on a conversation in familiar face-to-face situations;	The extent to which an individual can understand and use the oral and written language that appears in school textbooks and in discussions about the concepts embedded in school subjects.
Developed by the vast majority of native speakers by the time they enter formal schooling at age 5 or 6; phonology and fluency, in particular, reach a pleateau with minimal further development after age 5/6;	Develops together with conversational fluency in the early years but beomes differentiated from everyday spoken language as a result of exposure to more abstract concepts and printed text in the home and school; continues to develop through the school years and beyond;
Involves high-frequency words and expressions as well as relatively common grammatical constuctions;	Involves knowledge of low-frequency vocabulary and grammatical structures that are seldom used in everyday conversation (e.g., the passive voice);
Meaning is supported by facial expressions, gestures, eye contact, intonation, and the immediate enviroment;.	Meaning is made explicit through the language itself; teachers use specific instructional strategies (e.g., use of visuals, demonstrations, group work) to help students understand and use the language of schooling.

Table 1. Differences between conversational fluency and school language proficiency

Many researchers have identified similar patterns of difference between conversational and school language. Pauline Gibbons (1991), for example, expressed the distinction between what she calls *playground language* and *classroom language* as follows:

> This playground language includes the language which enables children to make friends, join in games and take part in a variety of day-to-day activities that develop and maintain social contacts. It usually occurs in face-to-face contact, and is thus highly dependent on the physical and visual context, and on gesture and body language. …
> But playground language is very different from the language that teachers use in the classroom, and from the language that we expect children to learn to use. The language of the playground is not the language associated with learning in mathematics, or social studies, or science. The playground situation does not normally offer children the opportunity to use such language as: *if we increase the angle by 5 degrees, we could cut the circumference into equal parts*. Nor does it normally require the language associated with the higher-order thinking skills, such as hypothesizing, evaluating, inferring, generalizing, predicting or classifying. (p. 3)

Teaching the Language of Mathematics

From an instructional perspective, the relationship between language and mathematics is two-way and reciprocal. Mathematical knowledge is developed through language, and language can and should be developed through mathematics instruction. Because mathematical concepts and operations are embedded in language, the specialized vocabulary of mathematics and the language structures of mathematical discourse must be modeled by the teacher and taught explicitly if students are to make strong academic progress in mathematics. Equally important, however, is the fact that in teaching mathematics we are also developing and reinforcing students' general academic language proficiency.

Consider the language teaching possibilities in the following mathematics word problem:

Is 3 + 8 greater than 10, equal to 10, or less than 10? Explain.

Students will learn not only the specific meanings of the terms *greater than*, *equal to*, and *less than*, but this particular mathematics problem also gives the teacher an opportunity to teach students the general concept of comparatives and the typical conventions for forming comparatives (e.g., *great, greater*). The fact that not all comparatives take exactly this form can also be taught in relation to *less, lesser, least*. Finally, the meaning of the word *explain* can be taught (e.g., describe, tell about, tell why you think so) and related to its use in other subject areas (e.g., science).

The interdependence of language and mathematics was addressed in a very significant report published by the Council of the Great City Schools (CGCS) (2016) entitled *A Framework for Re-envisioning Mathematics Instruction for English Language Learners*. The framework highlights the importance of enabling *all* students, including ELLs and students with language-related needs, to engage with complex grade-appropriate mathematical concepts and to solve real-world problems: “If students are provided with productive opportunities to engage in rigorous mathematics instruction, high cognitive demand tasks, and discussions, they will build both understanding of complex mathematical concepts as well as procedural fluency” (p. 7).

Furthermore, when students “are encouraged and taught how to communicate their mathematical understanding and reasoning, their mathematical learning will serve to reinforce and advance their development of English proficiency” (p. 7).

The framework emphasizes that instruction must reflect high expectations for all students and promote what it calls *agency* and *authority* among students.

> Agency is defined as *the student’s capacity and willingness to engage mathematically* and authority is defined as *the recognition for being mathematically capable*. Both agency and authority are built through student’s engagement in rigorous mathematical tasks and discussions that require them to conjecture, explain, construct mathematical arguments, and build on one another’s ideas. (p. 4)

In supporting students’ participation in mathematical tasks and discussions, teachers should avoid using only simplified language and instead model mathematical thinking and the use of precise mathematical language: “Teachers need to model the practice of making precise claims and support students in using increasingly more precise ways of describing mathematical situations” (p. 6). The goal is to expand students’ everyday language into the discourse of mathematical reasoning:

> Finally, in considering the complex interaction between language and learning mathematics, students’ everyday language and experiences should be understood and approached as resources, not as obstacles. The home language of students and informal ways of talking are assets for reasoning mathematically and provide a springboard teachers can use to develop academic language and support mathematical understanding. (p. 6)

In short, for early-stage ELLs and bilingual students, use of their home language for tasks and discussions in pairs or groups is fully appropriate. Engagement is key and if students can engage more effectively in their home language than in English, they should be encouraged to do so.

Mathematical Engagement: The Key to the Development of Expertise

In discussing the causal relationship between literacy engagement and literacy achievement, American cognitive psychologist, John Guthrie (2004), pointed out that in all spheres of life (e.g., driving a car, playing chess, gourmet cooking, and so on), participation is essential to the development of proficiency. He noted that “certainly some initial lessons are valuable for driving a car or typing on a keyboard, but expertise spirals upward mainly with engaged participation” (p. 8). The extensive research documenting the positive effects of literacy engagement on reading comprehension aligns with a more general set of findings regarding the benefits of active engagement on all aspects of academic performance. Boykin and Noguera (2011) point out that a “growing amount of research points to such engagement as particularly linked to favorable learning outcomes for minority students who have been placed at risk for academic failure” (p. 42). The CGCS framework similarly emphasizes the centrality of active engagement in *doing* mathematics for the development of expertise.

The framework outlined in Figure 2 extends the “Literacy Engagement” framework proposed by Cummins (e.g., Cummins & Early, 2011) to the sphere of mathematics. It specifies four broad instructional dimensions that are critical in enabling students to engage actively with mathematical reasoning and discussion from an early stage of their schooling. Mathematical engagement will be enhanced when:

- Teachers scaffold students’ ability to understand and apply mathematical concepts including the language used to express these concepts;
- Mathematics instruction connects to students’ lives and imaginations by activating their background knowledge and stimulating their curiosity and interest;
- Instruction affirms students’ sense of academic competence by enabling them to showcase their mathematical understanding and accomplishments;
- Mathematical concepts are reinforced across the curriculum and integrated with a focus on developing language awareness in other subjects such as science and social studies.

These instructional dimensions are often interdependent insofar as the same activity or project can simultaneously scaffold meaning, connect to students' lives, affirm their identities, and expand their mathematical knowledge across the curriculum. Instructional applications of this framework are discussed in the following sections.

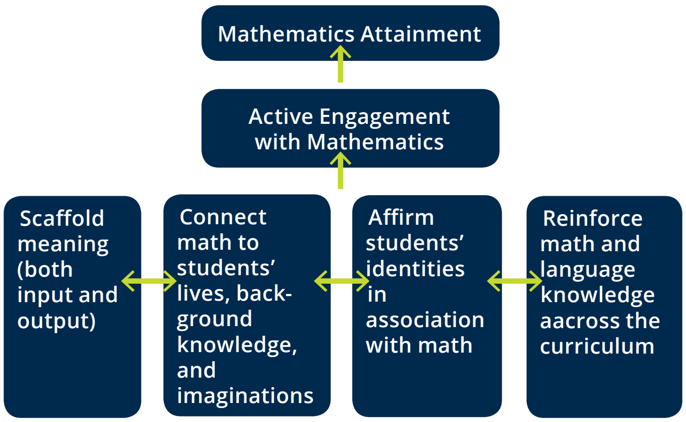

Figure 2. Mathematical Engagement Framework

SCAFFOLD MEANING

Scaffolding refers to the process of purposefully modifying classroom interactions in order to increase students' comprehension of content or language. Comprehension is facilitated when there are multiple routes to the meaning in addition to the language itself. Mathematics instruction is inherently multimodal because meaning is represented by symbols and images in addition to written and oral language. Maximizing this multimodality in mathematics instruction is particularly important in facilitating ELLs' participation in mathematics discourse. Additional strategies for instructional scaffolding are presented below.

- ***Demonstration/modeling*** For example, teachers can take students through a word problem in math demonstrating the kind of thinking that helps students understand and solve the problem.

- ***Use of hands-on manipulatives, tools, and technology*** Manipulatives such as counters and blocks enable students to carry out a mathematical operation, literally with their hands, and actually see the concrete results of this operation. Online manipulatives also develop this understanding. Measuring tools such as plastic rulers and protractors, as well as online tools, further support conceptual understanding, skills, and problem solving.

- ***Whole-class and small-group project work*** Working either as a whole class or in groups or pairs, students can engage with real-life or simulated projects that require application of a variety of mathematical skills. Díaz-Rico and Weed (2002) give as an example a project in which students are told the classroom needs to be re-carpeted. They first estimate the area, then check their estimates with measuring tools. Working in groups, students could also calculate the cost of floor covering using prices for various types of floor covering obtained from a local catalog.

- ***Use of visuals*** Visuals enable students to "see" the basic concept we are trying to teach much more effectively than if we rely only on words. Once students have the concept, they are much more likely to be able to figure out the meaning of the words we use to talk about it. Graphic organizers that are useful for teaching math vocabulary include: *Frayer Model, Word Chart, Word Web, and Word Map.*

- ***Language clarification*** This category includes a variety of strategies and language-oriented activities that clarify the meanings of new words and concepts. Teachers can modify their language to students by *paraphrasing ideas and explaining new concepts and words.* Important vocabulary can be repeated and recycled as part of the paraphrasing of ideas.

- ***Dramatization/Acting out*** For early-stage ELLs, *Total Physical Response*, where students act out commands, can be highly effective. Math calculations can be embedded in the commands that students act out. For example, students can progress from fully acting out the command *"Take 5 steps forward and 2 steps backward"* to calculating in their heads that they need only take 3 steps forward to reach the destination. Additionally, the meanings of individual words can be demonstrated through *gestures* and *pantomime*.

CONNECT TO STUDENTS' LIVES

The more connections we can make both to students' experiences and interests and to other areas of the curriculum, the more relevance mathematics is likely to assume in students' minds and lives. This, in turn, will result in more powerful learning of mathematics. Three types of background knowledge are relevant to consider in teaching mathematics:

1. Knowledge of mathematics concepts, facts, and skills that students have previously learned;

2. Knowledge of the world that students have acquired through their prior experiences;

3. Knowledge of the world that students have acquired through secondary sources such as books, television, movies, and the Internet.

The potential to increase student engagement though projects that connect math to students' lives is illustrated in a long-running international project coordinated by California educator Kristin Brown entitled *Connecting Math to Our Lives*. The project aimed to enable students to investigate how they could use math to analyze issues of importance to their society and take action to promote greater equity in their schools or communities. One of the projects focused on family purchases in different countries. Students collaborated with partner classes in different countries to collect data on prices and wages in their communities.

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AFFIRM IDENTITY

When students feel they are "good" at a particular subject, they are likely to be more enthusiastic in engaging with that subject. An effective strategy for getting students engaged in "doing mathematics" is to enable them to use mathematics to generate new knowledge by carrying out surveys on topics related to their lives or interests (see Coelho, 2012). Students can then analyze the data they have gathered using a variety of tools (e.g., calculators, spreadsheets), and they can present their findings on graphs, charts, and tables. Language survey projects are particularly suitable in multilingual school contexts because they enable students to discover more about themselves individually and collectively with respect to their linguistic talents and experiences. This kind of project can be carried out, with different degrees of sophistication and complexity, by students of any age. For example, primary grade students could use a colored marker to fill in squares of a simple chart, such as the one below, representing the home languages spoken by members of the class. After the data have been collected, a variety of grade-appropriate math activities can be carried out. Additionally, students could survey their parents and compare the data intergenerationally.

Arabic														
Cantonese														
English														
Mandarin														
Spanish														
Tamil														
Turkish														
Urdu														

Cummins and Early (2015) summarize one such survey in which Grade 5 students learned data management concepts and procedures by carrying out a survey of the multilingual nature of their entire school. The students not only generated knowledge about their own individual and collective linguistic talents but also affirmed the social legitimacy and the intellectual accomplishment that their multilingualism represented within the school and broader society. The academic engagement that this kind of identity-affirming project generated is reflected in the teacher's observation that students did not want to go out for recess because they were so busy creating pie charts, bar graphs, and other ways of representing their data.

Students' sense of accomplishment in using math to carry out these projects contributes to the development of what the CGCS (2016) termed *authority*—the development of an identity of confidence and competence in mathematics.

REINFORCE MATHEMATICS ACROSS THE CURRICULUM

Clearly, mathematical relationships and concepts are infused implicitly in most curricular subjects. For example, all kinds of social realities are expressed in mathematical terms (e.g., proportion of citizens who speak different languages or who fall into different income categories). Mathematics is also intrinsic to our understanding of scientific issues and concepts. For example, in assessing the scientific evidence for climate change, students could use publicly available data to examine changes over time in the number of severe weather events or they could investigate changes in average home insurance costs in different locations associated with the increase in severe weather events. Coordination of inquiry projects between mathematics and other areas of the curriculum would require teachers either individually or collectively within a school to identify topics where math and other curricular areas can be productively connected.

Teachers can also link the teaching of mathematics to language arts by drawing students' attention systematically to features of math vocabulary or typical discourse patterns in mathematics problems. One simple way of extending students' vocabulary and awareness of how academic English works is to explore the word families of common mathematical terms. For example, the word family of *multiply* would include the nouns *multiplication, multiple, multiplicity,* and the adjective *multiple*. The word family of *divide* includes the nouns *division* and *dividend* and the adjectives *divisive* and *divided*. Some of these words could then be reinforced in other curricular areas (e.g., social studies or discussion of current events). Students could also volunteer the translation equivalents of these words in their own languages and discuss cognate relationships between English and some of their own languages (e.g., multiplication and *multiplicación* in Spanish). Students could also research and generate other words in mathematics and other subjects that include the suffix -*tion*, which signifies an abstract noun usually formed by adding the suffix to a verb (e.g., *multiply*).

Conclusion

In order to teach mathematics effectively to all students and to help newcomer students and other ELLs to catch up academically as quickly as possible, we need to get students actively engaged in "doing mathematics."

Students are much more likely to get excited about a subject such as mathematics when they see its relevance to their own lives and when they understand how mathematics can provide powerful insights into social and physical realities. Students' understanding of academic content across the curriculum will be enhanced when teachers look for possibilities to integrate mathematics with the teaching of other subjects and also when they focus on the language of mathematics as a way of developing students' growing awareness of how academic language works.

In **enVision**® Mathematics © 2020, specific suggestions for language support in mathematics are provided throughout the core program and in the Language Support Handbook.

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