TOPIC 3
Mathematical Modeling in 3 Acts: The Express Lane

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Video

Lesson Overview

FOCUS

Objective

Students will be able to:

- ✓ Use mathematical modeling to represent a problem situation and to propose a solution.
- ✓ Test and verify the appropriateness of their math models.
- ✓ Explain why the results from their mathematical models might not align exactly with the problem situation.

Essential Understanding

Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly.

COHERENCE

Earlier in this topic, students:

- Interpreted statements that use function notation.

In this lesson, students:

- Develop a mathematical model to represent and propose a solution to a problem situation.

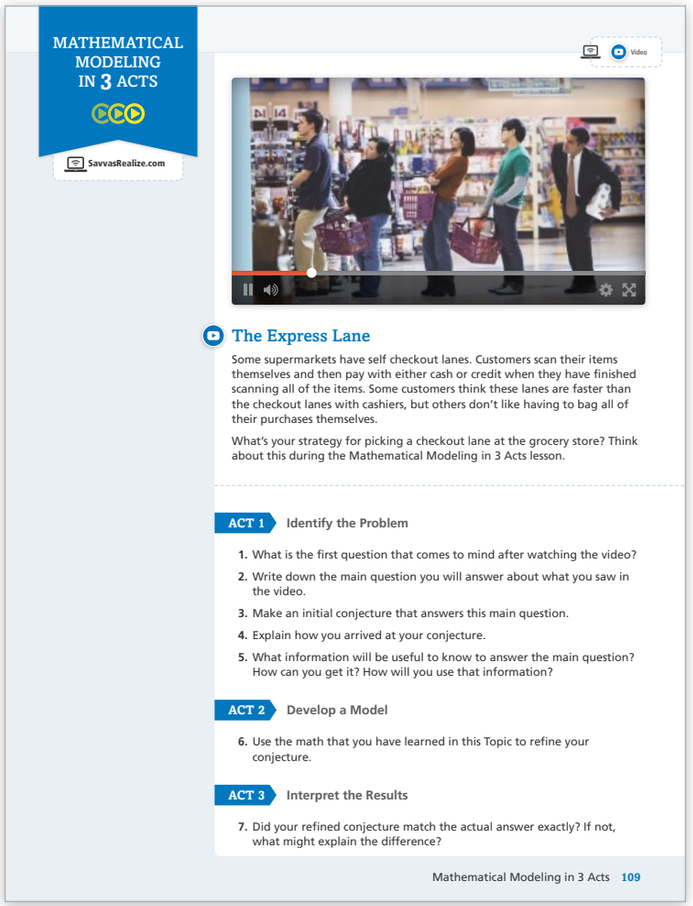
Later in this topic, students will:

- Find and analyze lines of best fit for scatter plots.

RIGOR

This mathematical modeling lesson focuses on application of both math content and math practices and processes.

- Students draw on their understanding of concepts related to linear models to develop a representative model.
- Students apply their mathematical model to test and validate its applicability to similar problem situations.


MATHEMATICAL MODELING IN 3 ACTS

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The Express Lane

Some supermarkets have self checkout lanes. Customers scan their items themselves and then pay with either cash or credit when they have finished scanning all of the items. Some customers think these lanes are faster than the checkout lanes with cashiers, but others don't like having to bag all of their purchases themselves.

What's your strategy for picking a checkout lane at the grocery store? Think about this during the Mathematical Modeling in 3 Acts lesson.

ACT 1 Identify the Problem

1. What is the first question that comes to mind after watching the video?
2. Write down the main question you will answer about what you saw in the video.
3. Make an initial conjecture that answers this main question.
4. Explain how you arrived at your conjecture.
5. What information will be useful to know to answer the main question? How can you get it? How will you use that information?

ACT 2 Develop a Model

6. Use the math that you have learned in this Topic to refine your conjecture.

ACT 3 Interpret the Results

7. Did your refined conjecture match the actual answer exactly? If not, what might explain the difference?

Mathematical Modeling in 3 Acts 109

Mathematics Overview

The Importance of Modeling

Rather than presenting a problem with all the necessary information, diagrams, and graphs in advance, this task also asks students to participate in the formulation of a problem.

Students decide what questions are interesting, what quantities are important, and how they could access information. They choose which math concepts are relevant to the task, using approximations or assumptions in the solution when necessary.

Finally, the task includes an actual, real-world answer. Students discuss possible sources of error inherent in using math to model real-world situations. Then they evaluate the usefulness of their models in the situation, improving them if necessary.

Applying Math Practices

Model With Mathematics

To solve the problem presented, students identify variables and the relationship among them, develop a model that represents the situation, and use the model to propose a solution. Students interpret their solutions and propose explanations for why their answer may not match the real-world answer.

Other Practices

Students also engage in sense-making, perseverance, and abstract and quantitative reasoning as they complete the task. In testing their models, students look for patterns in their models and use that structure to find a solution.

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The Express Lane

In this mathematical modeling task, students will explore and apply concepts related to collecting and using data to write a model. Students will analyze two lines at the grocery store for which data could be collected about several variables. They will use this data to create a linear model in order to make a decision. To do so, they apply concepts that they study in Topic 3.

ACT 1 The Hook

Play the video. The video shows two lines in a grocery store—an express lane and a regular lane. The express lane has 8 people with baskets while the regular lane has 3 people with carts.

After the question brainstorming, present to students the Main Question they will be tasked with answering. Remind students to write down their questions and conjectures.

MAIN QUESTION

Should you choose the express lane or the regular lane?

ACT 2 Modeling With Math

Think about the task. Ask students the variables they think affect how long it takes to get through the grocery store line. Then have them rank those variables by importance. Possible variables include total number of items in the lane, whether there is an employee bagging groceries, the payment method used, and whether the cashier is talkative.

Reveal the information. The second part of the video shows the number of customers in each lane and the number of items each customer is going to purchase. You can reveal a data table and scatter plot with information collected in a grocery store. Alternatively, you can have students collect their own data.

What's the connection? Give students time to struggle as they connect their ideas to what they learned in this topic. Suggest that they select and use a data display in order to see how the variables might be related. Which type of function best models the situation? How can students use the model they choose to make a decision?

INTERESTING MOMENTS WITH STUDENTS

Look for broad trends in students' first graph of the model. Do your students know that a graph supports a general claim about the relationship between two quantities? Do they have a sense of what the *y*-intercept should be? If they think 0 items will take 0 seconds, then the eventual model will be even more interesting.

Necessary Information

Express Lane

Customer 1: 3 items
Customer 2: 8 items
Customer 3: 4 items
Customer 4: 10 items
Customer 5: 5 items
Customer 6: 12 items
Customer 7: 8 items
Customer 8: 6 items

Regular Lane

Customer 1: 43 items
Customer 2: 21 items
Customer 3: 28 items

There are no unusual delays that cause one line to take much longer.

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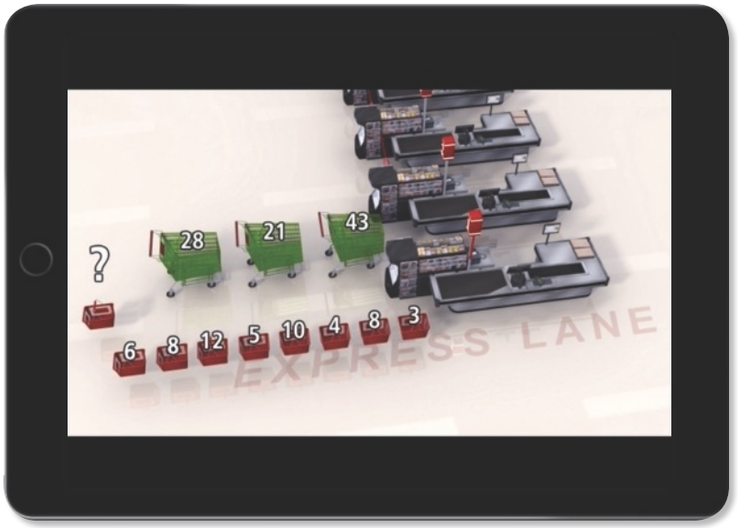
ACT 3 The Solution

Discuss the answer. The final video shows how long each line takes. Offer praise to the students whose conjectures are closest to the actual answer.



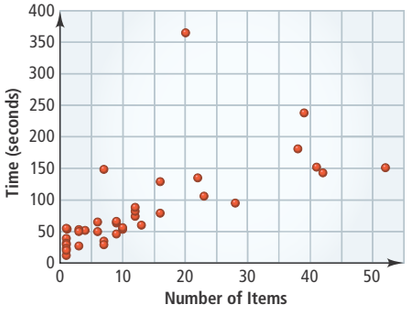
MAIN QUESTION ANSWER

The regular lane is about 3 minutes faster.

Do the "post-game" analysis. Ask students why the actual answer may differ from the mathematical answer. Someone may pay with a check, or a price check may occur. Cashiers also work at different speeds.

ONE POSSIBLE SOLUTION

Let x represent the number of items and y represent the time the transaction takes in seconds. Make a scatter plot.



The points appear to form a linear pattern. Using linear regression, the data can be modeled by the equation $y = 3.45x + 39.15$. On average, an item takes 3.45 seconds to scan, and the other elements of the transaction take an average of 39.15 seconds.

The regular lane will take 3.45(92) + 39.15(3) seconds, or 7:15. The express lane will take 3.45(56) + 39.15(8) seconds, or 8:26.

The express lane will be faster.

INTERESTING MOMENTS WITH STUDENTS

It's important to look at the data for potential issues. Students may mention that the small sample size may not be representative of all grocery store lines. They can make a list of the variables that should be considered in an improved model. Applaud these observations and invite students to collect their own data.

Play up the discovery. It is helpful to consider not only the number of people in line but also how many items each person has and other attributes of the cashiers and customers. By using math, we've unlocked some of those secrets.

SEQUEL

As students finish, ask them to propose a general rule for when they should choose the express lane. [Answers will vary. Look for a strategy that takes both scanning time and other time into account and could be used relatively quickly and without a calculator.]